SOME RESULTS ON HYPERK-ALGEBRAS

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ABSTRACT. In hyper$K$-algebras, the notion of a bounded hyper$K$-algebra and a homomorphism is introduced, and some properties related with a (weak) hyper$K$-ideal are investigated. The zero condition in a hyper$K$-algebra is considered, and then it is showed that every hyper$K$-algebra with the zero condition can be extended to a bounded hyper$K$-algebra.

1. Introduction The hyper algebraic structure theory was introduced in 1934 [7] by Marty at the 8th congress of Scandinavian Mathematiciens. Since then many researchers have worked on this area. Imai and Iseki in 1966 [3] introduced the notion of a BCK-algebra. Recently Jun et al. [6] applied the hyperstructures to BCK-algebras and introduced the concept of a hyperBCK-algebra which is a generalization of a BCK-algebra. Then Borzoei et al. [1] defined the notion of a hyper$K$-algebra. For background and notations we follow Borzoei et al. [1]. In this paper we introduced the notion of a bounded hyper$K$-algebra and a homomorphism of hyper$K$-algebras, and then we investigate some related results. We also consider the zero condition in hyper$K$-algebras. We show that every hyper$K$-algebra with the zero condition can be extended to a bounded hyper$K$-algebra.

2. Preliminaries

Definition 2.1 ([1], Definition 3.1). By a *hyper$K$-algebra* we mean a non-empty set H endowed with a hyperoperation "$\circ$" and a constant 0 satisfying the following conditions:

(HK1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK3) $x < x$,
(HK4) $x < y$ and $y < x$ imply $x = y$,
(HK5) $0 < x$,

for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a < b$.

Example 2.2 ([1], Example 3.2). (i) Define the hyper operation "$\circ$" on $H = [0, +\infty)$ as follows:

$$x \circ y := \begin{cases} [0, x] & \text{if } x \leq y \\ (0, y] & \text{if } x > y \neq 0 \\ \{x\} & \text{if } y = 0 \end{cases}$$

for all $x, y \in H$. Then $(H, \circ, 0)$ is a hyper$K$-algebra.

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(ii) Let $H = \{0, a, b\}$. Consider the following table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Then $(H, \circ, 0)$ is a hyper$K$-algebra.

(iii) Let $H = \{0, 1, 2\}$. Consider the following table:

$\circ$	0	1	2
0	$\{0\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$
1	$\{1\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$
2	$\{2\}$	$\{2\}$	$\{0, 1, 2\}$

Then $(H, \circ, 0)$ is a hyper$K$-algebra.

Theorem 2.3 ([1], Theorem 3.7). *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be two hyper$K$-algebra such that $H_1 \cap H_2 = \{0\}$ and $H = H_1 \cup H_2$. Then $(H, \circ, 0)$ is a hyper$K$-algebra, where the hyper operation "$\circ$" on H is defined by:*

$$x \circ y = \begin{cases} x \circ_1 y & \text{if } x, y \in H_1 \\ x \circ_2 y & \text{if } x, y \in H_2 \\ \{x\} & \text{otherwise,} \end{cases}$$

for all $x, y \in H$, and we denote it by $H_1 \oplus H_2$.

Theorem 2.4 ([1], Theorem 3.9). *Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be hyper$K$-algebras and $H = H_1 \times H_2$. We define a hyperopration "$\circ$" on H as follows,*

$$(a_1, b_1) \circ (a_2, b_2) = (a_1 \circ_1 a_2, b_1 \circ_2 b_2)$$

for all $(a_1, b_1), (a_2, b_2) \in H$, where for $A \subseteq H_1$ and $B \subseteq H_2$ by (A, B) we mean

$$(A, B) = \{(a, b) : a \in A, b \in B\}, \quad 0 = (0_1, 0_2)$$

and

$$(a_1, b_1) < (a_2, b_2) \Leftrightarrow a_1 < a_2 \text{ and } b_1 < b_2.$$

Then $(H, \circ, 0)$ is a hyper$K$-algebra, and it is called the hyper$K$-product of H_1 and H_2.

Definition 2.5 ([1], Definition 4.1). Let I be a non-empty subset of a hyper$K$-algebra $(H, \circ, 0)$. Then I is called a *weak hyper$K$-ideal* of H if

(Id1) $0 \in I$,

(Id2) $x \circ y \subseteq I$ and $y \in I$ imply that $x \in I$ for all $x, y \in H$.

Definition 2.6 ([1], Definition 4.4). Let I be a non-empty subset of a hyper$K$-algebra $(H, \circ, 0)$. Then I is said to be a *hyper$K$-ideal* of H if

(Id1) $0 \in I$,

(Id3) $x \circ y < I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

Note that every hyper$K$-ideal is a weak hyper$K$-ideal (see [1, Proposition 4.6]).

Definition 2.7 ([1], Definition 4.11). Let $(H, \circ, 0)$ be a hyper$K$-algebra and let S be a subset of H containing 0. If S is a hyper$K$-algebra with respect to the hyperoperation "$\circ$" on H, we say that S is a *hyper$K$-subalgebra* of H.

Theorem 2.8 ([1], Theorem 4.12). *Let S be a non-empty subset of a hyper$K$-algebra*

$(H, \circ, 0)$. *Then* S *is a hyper*K*-subalgebra of* H *if and only if* $x \circ y \subseteq S$ *for all* $x, y \in S$.

3. Bounded hyper$K$-algebras

Definition 3.1. Let $(H, \circ, 0)$ be a hyper$K$-algebra. If there exists an element $e \in H$ such that $x < e$ for all $x \in H$, then H is called a *bounded hyper*K*-algebra* and e is said to be the *unit* of H.

Note that (HK4) implies that the unit of H is unique.

Example 3.2. (i) Let $(X, *, 0)$ be a bounded BCK-algebra. Define the hyper operation "$\circ$" on X as follows:

$$x \circ y = \{x * y\}, \quad \forall x, y \in X.$$

Then $(X, \circ, 0)$ is a bounded hyper$K$-algebra.

(ii) The hyper$K$-algebra $(H, \circ, 0)$ in Example 2.2(i) is not bounded, because if $a \in H$ is unit, then $(a+1) \circ a = (0, a]$. Thus $0 \notin (a+1) \circ a$, i.e., $a + 1 \nless a$.

(iii) In Example 2.2(ii), H is bounded and $b \in H$ is unit.

(iv) The hyper$K$-algebra $(H, \circ, 0)$ in Example 2.2(iii) is bounded and $2 \in H$ is unit.

Proposition 3.3. *Let* H_1 *and* H_2 *be two bounded hyper*K*-algebras. Then the hyper*K*-product* $H_1 \times H_2$ *of* H_1 *and* H_2 *is also bounded.*

Proof. Let $e_1 \in H_1$ and $e_2 \in H_2$ be units and $(x, y) \in H_1 \times H_2$. Then $x < e_1$ and $y < e_2$ and so $(x, y) < (e_1, e_2)$. Therefore $H_1 \times H_2$ is bounded and (e_1, e_2) is its unit. □

The following example shows that if H_1 and H_2 are two bounded hyper$K$-algebras, then $H_1 \oplus H_2$ may not be bounded. For the notation $H_1 \oplus H_2$, we follow Borzoei [1].

Example 3.4. Let H_1 and H_2 be hyper$K$-algebras as in Examples 2.2(ii) and 2.2(iii) respectively. Then H_1 and H_2 are bounded, while $H_1 \oplus H_2$ is not bounded.

Definition 3.5. Let H be a hyper$K$-algebra. If $0 \circ x = \{0\}$ for all $x \in H$, then we say that H satisfies the *zero condition.*

Example 3.6. Let H be a hyper$K$-algebra as in Example 2.2(i). Then H satisfies the zero condition.

Theorem 3.7. *Let* $(H_1, \circ_1, 0)$ *be a hyper*K*-algebra, which satisfies the zero condition. Then* $(H_1, \circ_1, 0)$ *can be extended to a bounded hyper*K*-algebra.*

Proof. Let $e \notin H_1$ and $H = H_1 \cup \{e\}$. Define the hyper operation "$\circ$" on H as follows:

$$x \circ y = \begin{cases} \{e\} & \text{if } x = e,\ y \in H_1 \\ \{0\} & \text{if } x = e,\ y = e \\ \{0, x\} & \text{if } x \in H_1,\ y = e \\ x \circ_1 y & \text{if } x, y \in H_1, \end{cases}$$

for all $x, y \in H$. We show that $(H, \circ, 0)$ is a bounded hyper$K$-algebra and e is its unit.

(HK1): If $x, y, z \in H_1$, then by hypothesis (HK1) holds. Thus let at least one of x, y and z equal to e. If $x = e$ and $y, z \in H_1$, then

$$(e \circ z) \circ (y \circ z) = \{e\} \circ (y \circ z) = \{e\} < \{e\} = e \circ y.$$

If $z = e$ and $x, y \in H_1$, then

$$(x \circ e) \circ (y \circ e) = \{0, x\} \circ \{0, y\} = (0 \circ 0) \cup (0 \circ y) \cup (x \circ 0) \cup (x \circ y) < x \circ y.$$

If $y = e$ and $x, z \in H_1$, then

$$(x \circ z) \circ (e \circ z) = (x \circ z) \circ \{e\} = \{0\} \cup (x \circ z) < \{0, x\} = x \circ e.$$

If $x = z = e$ and $y \in H_1$, then since $0 < e$ we have

$$(e \circ e) \circ (y \circ e) = \{0\} \circ \{0, y\} = (0 \circ 0) \cup (0 \circ y) < \{e\} = e \circ y.$$

If $y = z = e$ and $x \in H_1$, then

$$(x \circ e) \circ (e \circ e) = \{0, x\} \circ \{0\} = (0 \circ 0) \cup (x \circ 0) < \{0, x\} = x \circ e.$$

If $x = y = z = e$, then

$$(e \circ e) \circ (e \circ e) = \{0\} \circ \{0\} < \{0\} = e \circ e.$$

(HK2): If $x, y, z \in H_1$, then (HK2) holds. Thus we let at least one of x, y, z equal to e. If $x = e$ and $y, z \in H_1$, then

$$(e \circ y) \circ z = \{e\} \circ z = \{e\} = \{e\} \circ y = (e \circ z) \circ e.$$

If $y = e$ and $x, z \in H_1$, then since H_1 satisfies the zero condition we get that

$$(x \circ e) \circ z = \{0, x\} \circ z = (0 \circ z) \cup (x \circ z) = \{0\} \cup (x \circ z) = (x \circ z) \cup \{0\} = (x \circ z) \circ e.$$

If $x = y = e$ and $z \in H_1$, then since H_1 satisfies the zero condition we have

$$(e \circ e) \circ z = \{0\} \circ z = \{0\} = \{e\} \circ e = (e \circ z) \circ e.$$

(HK3) Since $e \circ e = \{0\}$, thus $0 \in e \circ e$ and consequently $e < e$.

(HK4) and (HK5) are proved easily. Hence $(H, \circ, 0)$ is a hyper K-algebra. Moreover, since for any $x \neq e$, we have $x \circ e = \{0, x\}$, thus $x < e$. In other words $(H, \circ, 0)$ is bounded with unit e. □

4. Homomorphisms of hyper$K$-algebras

Definition 4.1. Let H_1 and H_2 be two hyper$K$-algebras. A mapping $f : H_1 \to H_2$ is said to be a *homomorphism* if

(i) $f(0) = 0$

(ii) $f(x \circ y) = f(x) \circ f(y), \quad \forall x, y \in H_1$.

If f is 1-1 (or onto) we say that f is a *monomorphism* (or *epimorphism*). And if f is both 1-1 and onto, we say that f is an *isomorphism.*

Example 4.2. Let H be as in Example 2.2(i) and $t \in \mathbb{R}^+$ be constant. Define

$$f : H \to H, \quad f(x) = tx, \quad \forall x \in H.$$

Then f is an isomorphism of hyper$K$-algebras. To do this, let $x, y \in H$ and $x \leq y$. Then $tx \leq ty$ and thus $f(x \circ y) = f([0, x]) = [0, tx] = tx \circ ty = f(x) \circ f(y)$. If $x > y \neq 0$, then $tx > ty$ and so

$$f(x \circ y) = f((0, y]) = (0, ty] = tx \circ ty = f(x) \circ f(y).$$

If $y = 0$, then

$$f(x \circ 0) = f(\{x\}) = tx = tx \circ t0 = f(x) \circ f(0).$$

Also $f(0) = 0$, consequently f is a homomorphism. Clearly f is onto and 1-1. Thus f is an isomorphism.

Theorem 4.3. *Let $f : H_1 \to H_2$ be a homomorphism of hyper$K$-algebras. Then*

(i) *If S is a hyper$K$-subalgebra of H_1, then $f(S)$ is a hyper$K$-subalgebra of H_2,*

(ii) *$f(H_1)$ is a hyper$K$-subalgebra of H_2,*

(iii) *If H_1 satisfies the zero condition, then so is $f(H_1)$,*

(iv) *If S is a hyper$K$-subalgebra of H_2, then $f^{-1}(S)$ is a hyper$K$-subalgebra of H_1,*

(v) *If I is a (weak) hyper$K$-ideal of H_2, then $f^{-1}(I)$ is a (weak) hyper$K$-ideal of H_1,*

(vi) Ker$f := \{x \in H_1 \mid f(x) = 0\}$ *is a hyper$K$-ideal and hence a weak hyper$K$-ideal of H_1,*

(vii) *If f is onto and I is a hyper$K$-ideal of H_1 which contains* Ker$f$, *then $f(I)$ is a hyper$K$-ideal of H_2.*

Proof. (i) Let $x, y \in f(S)$. Then there exist $a, b \in S$ such that $f(a) = x$ and $f(b) = y$. It follows from Theorem 2.8 that

$$x \circ y = f(a) \circ f(b) = f(a \circ b) \subseteq f(S)$$

so that $f(S)$ is a hyper$K$-subalgebra of H_2.

(ii) It is straightforward by (i).

(iii) If H_1 satisfies the zero condition, then $0 \circ x = \{0\}$ for all $x \in H_1$. Let $y \in f(H_1)$. Then there exists $a \in H_1$ such that $f(a) = y$. It follows that

$$0 \circ y = f(0) \circ f(a) = f(0 \circ a) = f(\{0\}) = \{0\}$$

so that $f(H_1)$ satisfies the zero condition.

(iv) Since $0 \in S$, we have $f^{-1}(0) \subseteq f^{-1}(S)$. Since $f(0) = 0$, so $0 \in f^{-1}(0) \subseteq f^{-1}(S)$. Therefore $f^{-1}(S)$ is non-empty. Now let $x, y \in f^{-1}(S)$. Then $f(x)$, $f(y) \in S$. Thus $f(x \circ y) = f(x) \circ f(y) \subseteq S$ and so $x \circ y \subseteq f^{-1}(S)$, which implies that $f^{-1}(S)$ is a hyper$K$-subalgebra of H_1.

(v) Let I be a weak hyper$K$-ideal of H_2. Clearly $0 \in f^{-1}(I)$. Let $x, y \in H_1$ such that $x \circ y \subseteq f^{-1}(I)$ and $y \in f^{-1}(I)$. Then $f(x) \circ f(y) = f(x \circ y) \subseteq I$ and $f(y) \in I$. Since I is a weak hyper$K$-ideal, it follows from (Id2) that $f(x) \in I$, i.e., $x \in f^{-1}(I)$. Hence $f^{-1}(I)$ is a weak hyper$K$-ideal of H_1. Now let I be a hyper$K$-ideal of H_2. Obviously $0 \in f^{-1}(I)$. Let $x, y \in H_1$ such that $x \circ y < f^{-1}(I)$ and $y \in f^{-1}(I)$. Then there exist $t \in x \circ y$ and $z \in f^{-1}(I)$ such that $t < z$, i.e., $0 \in t \circ z$. Since $f(z) \in I$ and $0 \in t \circ z \subseteq (x \circ y) \circ z$, it follows that

$$0 = f(0) \in f((x \circ y) \circ z) = f(x \circ y) \circ f(z) \subseteq f(x \circ y) \circ I$$

so that $f(x) \circ f(y) = f(x \circ y) < I$. As $f(y) \in I$ and I is hyper$K$-ideal, by using (Id3) we have $f(x) \in I$, i.e., $x \in f^{-1}(I)$. Hence $f^{-1}(I)$ is a hyper$K$-ideal of H_1.

(vi) First we show that $\{0\} \subseteq H_2$ is a hyper$K$-ideal. To do this, let $x, y \in H_2$ be such that $x \circ y < \{0\}$ and $y \in \{0\}$. Then $y = 0$ and so $x \circ 0 = x \circ y < \{0\}$. Therefore there exists $t \in x \circ 0$ such that $t < 0$. Thus $t = 0$, and consequently $0 \in x \circ 0$, i.e., $x < 0$, which implies that $x = 0$. This shows that $\{0\}$ is a hyper$K$-ideal of H_2. Now by (v), Ker$f = f^{-1}(\{0\})$ is a hyper$K$-ideal of H_1.

(vii) Since $0 \in I$, we have $0 = f(0) \in f(I)$. Let x and y be arbitrary elements in H_2 such that $x \circ y < f(I)$ and $y \in f(I)$. Since $y \in f(I)$ and f is onto, there are $y_1 \in I$ and $x_1 \in H_1$ such that $y = f(y_1)$ and $x = f(x_1)$. Thus

$$f(x_1 \circ y_1) = f(x_1) \circ f(y_1) = x \circ y < f(I).$$

Therefore there are $a \in x_1 \circ y_1$ and $b \in I$ such that $f(a) < f(b)$. So $0 \in f(a) \circ f(b) = f(a \circ b)$, which implies that $f(c) = 0$ for some $c \in a \circ b$. It follows that $c \in$ Ker$f \subseteq I$ so that $a \circ b < I$. Now since I is a hyper$K$-ideal of H_1 and $b \in I$, we get $a \in I$. Thus $x_1 \circ y_1 < I$, which implies that $x_1 \in I$. Thereby $x = f(x_1) \in f(I)$, and so $f(I)$ is a hyper$K$-ideal of H_2. □

The following theorem is straightforward, and we omit the proof.

Theorem 4.4. Let $f : H_1 \to H_2$ be an epimorphism of hyper$K$-algebras. Then there is a one to one correspondence between the set of all hyper$K$-ideals of H_1 containing Ker$f$ and the set of all hyper$K$-ideals of H_2.

Lemma 4.5. *Let $f : H_1 \to H_2$ be a homomorphism of hyper$K$-algebras. If $x < y$ in H_1, then $f(x) < f(y)$ in H_2.*

Proof. If $x < y$ in H_1, then $0 \in x \circ y$ and so

$$0 = f(0) \in f(x \circ y) = f(x) \circ f(y).$$

Therefore $f(x) < f(y)$. □

Theorem 4.6. *Let $f : H_1 \to H_2$ be an epimorphism of hyper$K$-algebras. If H_1 is bounded, then H_2 is also bounded.*

Proof. Let e be the unit of H_1 and $y \in H_2$ be an arbitrary element. Then there exists $x \in H_1$ such that $f(x) = y$. Since $x < e$, by Lemma 4.5 we have $y = f(x) < f(e)$. Thus $f(e)$ is the unit of H_2 and H_2 is bounded. □

Theorem 4.7. *Let $f : H_1 \to H_2$ and $g : H_1 \to H_3$ be two homomorphisms of hyper$K$-algebras such that f is onto and* Ker$f \subseteq$ Ker$g$. *Then there exists a homomorphism $h : H_2 \to H_3$ such that $h \circ f = g$.*

Proof. Let $y \in H_2$ be arbitrary. Since f is onto, there exists $x \in H_1$ such that $y = f(x)$. Define $h : H_2 \to H_3$ by $h(y) = g(x)$, $\forall y \in H_2$. Now we show that h is well-defined. Let $y_1, y_2 \in H_2$ and $y_1 = y_2$. Since f is onto, there are $x_1, x_2 \in H_1$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Therefore $f(x_1) = f(x_2)$ and thus $0 \in f(x_1) \circ f(x_2) = f(x_1 \circ x_2)$. It follows that there exists $t \in x_1 \circ x_2$ such that $f(t) = 0$. Thus $t \in$ Ker$f \subseteq$ Ker$g$ and so $g(t) = 0$. Since $t \in x_1 \circ x_2$ we conclude that

$$0 = g(t) \in g(x_1 \circ x_2) = g(x_1) \circ g(x_2)$$

which implies that $g(x_1) < g(x_2)$. On the other hand since $0 \in f(x_2) \circ f(x_1) = f(x_2 \circ x_1)$, similarly we can conclude that $0 \in g(x_2) \circ g(x_1)$, i.e., $g(x_2) < g(x_1)$. Thus $g(x_1) = g(x_2)$, which shows that h is well-defined. Clearly $h \circ f = g$. Finally we show that h is a homomorphism. Let $y_1, y_2 \in H_2$ be arbitrary. Since f is onto there are $x_1, x_2 \in H_1$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Then

$$\begin{aligned} h(y_1 \circ y_2) &= h(f(x_1) \circ f(x_2)) = h(f(x_1 \circ x_2)) = (h \circ f)(x_1 \circ x_2) \\ &= g(x_1 \circ x_2) = g(x_1) \circ g(x_2) = (h \circ f)(x_1) \circ (h \circ f)(x_2) \\ &= h(f(x_1)) \circ h(f(x_2)) = h(y_1) \circ h(y_2). \end{aligned}$$

Moreover since $f(0) = 0$ and $g(0) = 0$, we conclude that

$$h(0) = h(f(0)) = (h \circ f)(0) = g(0) = 0.$$

Thus h is a homomorphism, ending the proof. □

Theorem 4.8. *Let $f : H_1 \to H_2$ be a monomorphism of hyper$K$-algebras. If H_2 is bounded with unit element e and $e \in$* Imf, *then H_1 is also bounded and $f^{-1}(e)$ is its unit.*

Proof. Let $x \in H_1$. Then $f(x) \in H_2$. Since H_2 is bounded we conclude that $f(x) < e$, and since $e \in$ Imf, we get that $e = f(a)$ for some $a \in H_1$. Thus $f(x) < f(a)$. Therefore $0 \in f(x) \circ f(a) = f(x \circ a)$. It follows that there exists $b \in x \circ a$ such that $f(b) = 0$. Hence $b = 0$, because f is 1-1. Thus $0 \in x \circ a$, i.e., $x < a$. Now since $a = f^{-1}(e)$, we conclude that

$x < f^{-1}(e)$, which shows H_1 is bounded with unit $f^{-1}(e)$. □

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