AN OPTIMAL DISPATCH PLANNING OF GUARDS TO COUNTER TO A SUICIDE BOMBER

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Abstract. Counter terrorism operations become one of the great concerns in the recent military affairs. We will propose an optimal planning method for dispatching of security personnel to protect the lives of citizen from suicide bombing. Some security staffs, such as police or military personnel, are dispatched to patrol a certain area during a designated period. The whole operation time is divided into small mission periods and the limited number of securities take turns on the patrol for each period. In each mission period, there come some citizens there. Each of the dispatched guards firstly patrols there to search for a doubtful person. When they detect the man, they communicate with each other and try to neutralize him. Under such a situation, we develop a dispatch planning tool so as to maximize the expected number of surviving guards and the citizens from detonation within each divided period.

1 Introduction Suicide bombing is one of the major attack methods in the recent terrorism incidents. According to the terrorism database [12] in the latest 40 years, more than 60 % of terrorism incidents happened after aircraft attacking to U.S. on September 11 and the bombing is one of the most popular attack methods. The bombing incidents include not only suicide bombing, but also concealed bombs in roadways or bombing in a stadium etc. Geographically, this kind of bombings have happened mainly in Middle East or South Asia, but now is expanding to South-East Asia, Africa and to all over the world. We must regard that the (suicide) bombing becomes common and may happen everywhere around us. Therefore we must develop some effective counter measures as soon as possible.

Government or public sectors of many countries have begun to study the legal counter measures. They reinforce the regulation of the anti-terrorism law or restrict international remittance or procurement of chemical materials that support the terrorism actions [9]. Coincident with those legal amendments, scientists also have begun to put their energy into developing standoff explosives-detection technologies [10]. In general, the situation awareness of bombings except suicide ones is very poor because the clues of bombing incident are too ambiguous to perceive. Or even if the suspicious is detected by the guards such as police or military personnel, they are hardly able to neutralize him because of their poor equipments.

National Research Council (NRC) has started developing counter techniques, such as detection systems for an explosive, to assist the project of the Defense Advanced Research Projects Agency (DARPA). NRC is also studying about elements of detection, concepts and scenarios of bombing from the scientific viewpoint. One of their primal scenarios is suicide bombing. To tackle it, the NRC panel investigates scientific parameter values; e.g. walking speed of a suicide bomber or sensor-detection range.

On the other hand, Operations Research has just started treating counter-terrorism problem. Kress [7] first treated the terrorism problem. He made a model that estimates

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the casualties caused by suicide bombing and applied it to real incidents in Israel. The real number of the casualties is not so different from the calculation. Before Kress, the number of casualties is counted not in the context of terrorism but in the context of accidents in military operations [4, 8]. After Kress, many studies about counter-terrorism have been published. Kaplan and Kress [6] extended the Kress's model and applied it to more realistic situations. They proposed the grid model and the plaza model to estimate the number of casualties and considered the optimal allocation of search sensors. Nie et al. [11] also optimized the allocation of detecting sensors in a square area. Berman [2] discusses the optimal pre-positioning of the governmental facilities against terrorism attack. In addition, the special issue about ongoing military operations and the counter-terrorism was published recently [1].

Most of those studies deal with the facility allocation problem for the counter-terrorism. In this paper, we propose a model applicable to more dynamic situation. Security sectors, such as police office or military troop, have responsibility to guard a certain facility during some periods. For the counter operation to suicide bombing, they make a plan to dispatch their staffs to the facility. We will call the facility Arena, as Kress named. Many people come, stay or leave there. As concrete examples, we can think of city hall, station or shopping center. Security staffs must patrol the Arena by turns for several periods, to each of which some guards are assigned. A suicide bomber may or may not appear in the period. In this setting, the number of the dispatched guards must be optimized.

The optimal decision may depend on many factors, such as the arriving probability of a suicide bomber within the period, the number of available guards, the detection capability of the guards, the area of the Arena or the number of people staying in the Arena. In this paper, we will optimize the number of guards to be dispatched for each designated period, considering those factors. The numbers are decided so as to maximize the expected total number of saved lives of guards and citizens until the end of the pre-planned periods.

In Section 2, we describe assumptions of our model. In Section 3, we formulate the planning model in a recursive form. In Section 4, we will show some numerical examples and analyze the properties of optimal dispatching strategy. In Section 5, we extend the model to a multi-Arena situation. In the last section, we summarize our results and describe the further extensions.

2 Assumptions and Notation For modeling the dispatching of guards, we describe some assumptions.

1. Security guards, such as police or military, consist of k persons, must patrol an Arena. The time horizon of their operations is divided into T mission periods and x out of survived staffs are dispatched to patrol the Arena in each period by turns.

2. Each mission period is numbered by $T, T-1, \cdots, 1, 0$, which represents the residual until the end of the operation. The 0-th period is the end of whole mission.

3. The number of citizen in the Arena during the t-th period is $c(t)$.

4. At most one suicide bomber comes to the Arena with probability λ.

5. The missions of the guards are to patrol and to save the lives of citizen in the Arena. If they can detect an arriving suicide bomber, they approach him/her to counter its aim. Each guard is assumed to have the detection capability p. that each guard can detect the suspicious with probability p.

6. When no guard can detect him, the suicide bomber detonates his bomb at the most effective point in the Arena. In that case, both the guards and the citizen there suffer severe damage.

7. If the patrolling guards can detect him, they fight him to prevent the detonation. The fight may end up in success or failure. The success means the capture or the death of the target without any detonation. In that case, some guards may be killed or wounded but no citizen suffers any damage.

8. The failure of the neutralization means the detonation. In the worst case, all guards are killed but the bomber is still acitve, he executes suicide bombing without hesitation at the most effective point in the Arena. In those failure cases, both guards and citizens suffer severe damage.

9. The assault battle ends in a short time. The numbers of citizen and guards are assumed to be the same value during the battle period.

We decide the optimal number of dispatching guards x in each mission period so as to maximize the total number of the survival until the end of the operation periods.

3 Dispatch Planning Model In this Section, we formulate a dispatch planning model by dynamic programming formulation. A solution is given as the optimal number of dispatching guards for each mission period. In Section **3.1**, we first focus on all possible events, which may happen during each mission period. In Section **3.2**, we classify the results of the battle between the guards and a bomber and calculate the success/failure probabilities. And in **3.3**, we show how to estimate the casualties when the bomb explodes in the Arena. Using those probabilities and the estimation, we formulate the recursive model in **3.4**.

3.1 All possible events in a mission period In any mission period, several guards, including zero, are dispatched to Arena. All possible events in the t-th period are depicted in Figure 1.

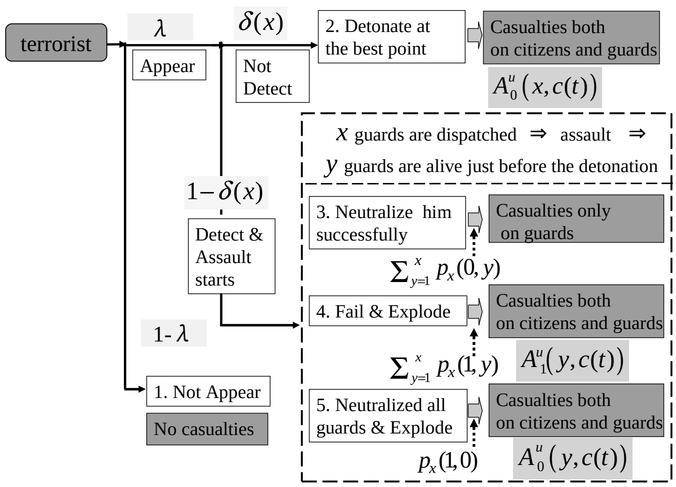

Figure 1 Possible events in a mission period at the Arena

All the possible events are classified into five cases. In the first case, a suicide bomber does not appear with probability $1-\lambda$. ¿From the second to the fifth case, a suicide bomber does come there. These cases are divided based on whether the guards can detect him or not, and the result of the battle. In the second case, no guard can detect the suicide bomber and he explodes his bomb at the most effective point. The guards do not find the bomber with probability $\delta(x)$. Assuming that each guard searches for the bomber independently, $\delta(x)$ is $(1-p)^x$. The expected number of casualties is estimated by $A_0^u(x, c(t))$. How to calculate the estimation is described in Section **3.3**. The upper index $u = c$ stands for the citizen side or $u = s$ for the security guard side and the lower index indicates that the explosion happens at the most effective point (0), or elsewhere (1) in the Arena.

From the third to the fifth case, the guards recognize the bomber and a battle takes place between them. In the following cases, y denotes the numbers of active guards. In the third case, the bomber is neutralized by the guards and the detonation is avoided. We assume that the battle is limited between the guards and the bomber and no citizen suffers any damage from the battle. We denote the probability that the fight starts with x guards and ends successfully with no active suicide bomber and y remaining guards by $p_x(0, y)$, which is derived in Section **3.2**. The first argument (0) means the number of active suicide bomber. In the fourth and the fifth cases, the battle also takes place but the guards cannot avoid the detonation in the end. The probability that the fight ends in failure with one active suicide bomber and y residual guards is $p_x(1, y)$. And the bomb explodes during the battle, the estimated number of casualties is denoted by $A_1^u(y, c(t))$ for the citizen ($u = c$) and the security team ($u = s$). On the estimation, we take the average of casualties weighted by the explosive point. In the fifth case, all the guards are defeated thoroughly ($y = 0$) and the bomber can explode his bomb at the best point. As we assumed that the sequential events end in a short time, all $c(t)$ citizens stay in the arena during the battle and cannot escape from the situation.

3.2 The results of the battle estimated by Lanchester model The terrorist may be neutralized or not in the end of the battle. He may push a button of the bomb at every moment during the battle. The result is stochastic. As the battle takes place between the small number of guards and a terrorist, we adopt the stochastic Lanchester model of square law [3] to proceed the attrition process. When the process obeys the square law, the kill

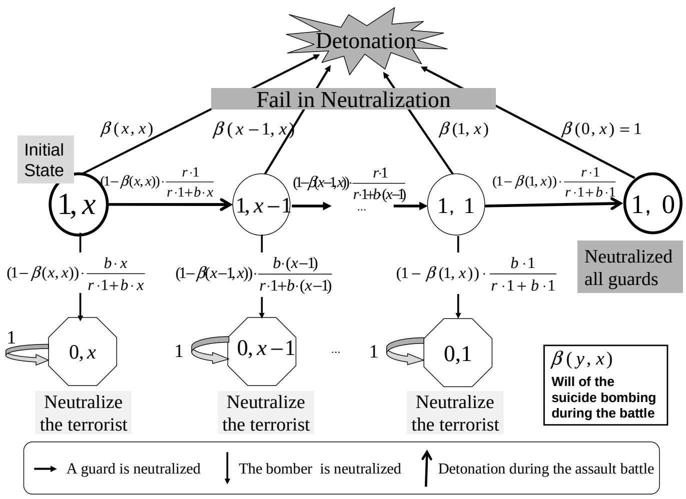

Figure 2 State transition during the assault battle

capability of fighters is linearly proportional to the remaining force size. Under these assumptions, the transition process during the battle is depicted in Figure 2. The numbers in each circle represent the number of bomber and active guards.

The battle starts from the state of one terrorist and x guards. From the left to the right, the number of the guards decreases one by one. Every downward arrow indicates the neutralization of the terrorist. Upward arrows mean the failure of the mission, or the explosion of the bomb. The parameters r, b are the kill rates of the suicide bomber and a guard, respectively. We assume that the bomber explodes his bomb with probability $\beta(y, x)$ when there are y active guards during the battle. We assume that the probability approaches 1 as the battle goes by or y decreases.

In numerical examples explained later, we use the following function,

$$\beta(y, x) = S^{\,y/x}. \tag{1}$$

The parameter S, $0 < S < 1$, represents the strength of bomber's urge or will for the explosion. It approaches convexly up to one at $y = 0$. We may need consulting with some psychological knowledge about the shape of the function.

When the attrition process during the battle obeys the square law of stochastic Lanchester model [3], the probability that x guards decrease by 1 is $r/(r + bx)$. On the other hand, the probability that a suicide bomber is neutralized is $bx/(r + bx)$.

The success probability, $p_x(0, y)$, is calculated by the simultaneous events that (1) the initial x guards are continuously defeated by a suicide bomber from x to $y + 1$, (2) but the suicide bomber is neutralized by y guards in the end, and (3) the bomber does not make an explosion through the battle. As the result, the probability is expressed as follows.

$$p_x(0, y) = \left[\prod_{n=y+1}^{x} (1 - \beta(n, x))\ \frac{r \cdot 1}{r \cdot 1 + b \cdot n}\right] (1 - \beta(y, x))\ \frac{b \cdot y}{r \cdot 1 + b \cdot y}\ . \tag{2}$$

On the other hand, the failure probability that the guards cannot stop the suicide bombing by y guards becomes as follows.

$$p_x(1, y) = \left[\prod_{n=y+1}^{x} (1 - \beta(n, x))\ \frac{r \cdot 1}{r \cdot 1 + b \cdot n}\right] \beta(y, x)\ . \tag{3}$$

In Eqs.(2) and (3), y can take any value from x to 1 and the value of the brackets equals one in the case of $y = x$.

3.3 Estimation of casualties by suicide bombing To calculate the casualties by a suicide bombing, we adopt the model of Kress [7]. In his model, the shape of the Arena is assumed to be circular. Then, the most effective point of the detonation is the center of the Arena. When the bombing happens, the people there are assumed to be distributed uniformly. Kress calculates the expected casualties by the following three steps.

At first, he divides the circular Arena into concentric rings. The width of each ring is assumed to be the diameter of human body. Then the Arena is assumed to be divided into M rings. He calculates the expected number of people standing in the m-th ring as μ_m. The number is proportional to the size of each ring. Secondly, the probability that there is no one between the bombing point and a person in the m-th ring, $\gamma(m)$, is calculated. The person certainly suffers damage if at least one fragment hits the person. Kress denotes the probability that a person in the m-th ring is hit by $P_H(m)$. By multiplying these values, he estimates the casualties in a circular Arena.

We make use of this estimation. When y active guards and c citizens are distributed uniformly and the detonation happens at the center of the Arena, the casualties are estimated by

$$A_0(y,c) = \sum_{m=1}^{M} \mu_m \cdot \gamma(m) \cdot P_H(m) \ . \tag{4}$$

By dividing $A_0(y,c)$ to two sides of the security and the citizen proportionally to their population y and c, we estimate the casualties on both sides as follows.

$$A_0^s(y,c) = A_0(y,c) \cdot \frac{y}{y+c}, \tag{5}$$

$$A_0^c(y,c) = A_0(y,c) \cdot \frac{c}{y+c}. \tag{6}$$

When the suicide bombing occurs at points other than the center, we modify the Kress's estimation geometrically and obtain the expected number of casualties, which are denoted by $A_1^s(y,c)$ and $A_1^c(y,c)$ for the security and the citizen. Please refer to Appendix A for detail.

3.4 Optimal Dispatching Plan To maximize the total number of survived guards and citizen until the end of the entire mission periods, we can calculate the optimal number of dispatching guards to the Arena by a dynamic programming formulation. Let $FE[t,k]$ be the maximal expectation of survived number of guards and citizens from the beginning of the t-th period to the end of whole mission periods. At that time, k guards are assumed to be available.

$$\begin{aligned} FE[t,k] &= (1-\lambda)(FE[t-1,k]+c(t)) \\ &+ \lambda \max_{0\le x\le k}\Big[\delta(x)(FE[t-1,k-A_0^s(x,c(t))]+c(t)-A_0^c(x,c(t))) \\ &+ (1-\delta(x))\Big\{\sum_{y=1}^{x} p_x(0,y)(FE[t-1,k-x+y]+c(t)) \\ &\qquad +\sum_{y=1}^{x} p_x(1,y)(FE[t-1,k-x+y-A_1^s(y,c(t))]+c(t)-A_1^c(y,c(t))) \\ &\qquad + p_x(1,0)(FE[t-1,k-x]+c(t)-A_0^c(0,c(t)))\Big\}\Big], \end{aligned} \tag{7}$$

$$FE[0,k] = k \ . \tag{8}$$

Each line of Eq.(7) corresponds to the five events, explained in Section **3.1**. The maximization included in the second line gives us the optimal number of dispatching guards in the t-th period. The computation starts from the last period or Eq.(8). At $t = 0$, the whole operation is over without any happening and all available guards are alive. So we can set the initial condition as Eq.(8) for all up to k.

To calculate $FE[t,k]$, we only need $FE[t-1,k]$ at the stage $t-1$. Because the second argument of $FE[t,\cdot]$ in the second and the fourth line of Eq.(7), such as $k-x+y-A_1^s(y,c(t))$ is not integer in general, we approximate $FE[t,z]$ to $(z-\lfloor z\rfloor)FE[t,\lceil z\rceil]+(\lceil z\rceil - z)FE[t,\lfloor z\rfloor]$ for real number z by interpolation. In the next section, we investigate those optimal dispatching plans by some numerical examples.

4 Numerical Examples We first consider a basic case and then compare some cases by changing some parameters.

4.1 Basic case We will calculate the case where there are 10 mission periods and 10 available guards. An Arena has the diameter of 30m. Each guard has the capability of detection probability $p = 0.5$ for a suicide bomber by using various standoff detection sensors or bomb dogs [13]. If every guard patrols in the arena independently, the non-detection probability by x guards is estimated as $\delta(x) = (1-p)^x$. Suicide bomber comes to an arena with rate $\lambda = 1/30$. His initial will for bombing is $S = 0.5$. As in Kress [7], an ordinary bombing makes 100 harmful fragments and scatters them all around from the bombing point. When the battle takes place, the kill rates of the bomber and each guard are set to be the same value, $r = b$. During each mission period, $c(t) = 30$ citizens stay in the arena.

Under these conditions, the optimal dispatching plan of guards is illustrated in Table 1

Table 1. The optimal number of dispatching guards (basic case)

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	6	6	6	7	7	7	7	7	7	7
9	6	6	6	6	6	6	7	7	7	7
8	5	5	5	5	6	6	6	6	7	7
7	5	5	5	5	5	5	5	6	7	7
6	4	4	4	5	5	5	6	6	6	6
5	4	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1

Table 2. The expectation of the survived guards(G) and the cumulative citizens(C) (basic case)

$k \setminus t$		10	9	8	7	6	5	4	3	2	1
10	G	8.9	9.0	9.1	9.2	9.3	9.4	9.6	9.7	9.8	9.9
	C	298.8	269.0	239.1	209.2	179.3	149.4	119.6	89.7	59.8	29.9
9	G	7.9	8.0	8.1	8.2	8.3	8.4	8.6	8.7	8.8	8.9
	C	298.8	268.9	239.1	209.2	179.3	149.4	119.5	89.7	59.8	29.9
8	G	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9
	C	298.7	268.8	239.0	209.1	179.3	149.4	119.5	89.7	59.8	29.9
7	G	6.1	6.2	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9
	C	298.6	268.8	238.9	209.1	179.2	149.4	119.5	89.7	59.8	29.9
6	G	5.1	5.2	5.3	5.3	5.4	5.5	5.6	5.7	5.8	5.9
	C	298.5	268.7	238.9	209.0	179.2	149.4	119.5	89.6	59.8	29.9
5	G	4.2	4.2	4.3	4.4	4.5	4.6	4.6	4.7	4.8	4.9
	C	298.4	268.6	238.8	208.9	179.1	149.3	119.4	89.6	59.7	29.9
4	G	3.3	3.4	3.4	3.5	3.6	3.6	3.7	3.8	3.9	3.9
	C	298.2	268.4	238.6	208.8	179.0	149.1	119.3	89.5	59.7	29.8
3	G	2.5	2.5	2.6	2.6	2.7	2.7	2.8	2.8	2.9	2.9
	C	297.8	268.0	238.3	208.5	178.7	149.0	119.2	89.4	59.6	29.8
2	G	1.6	1.7	1.7	1.7	1.8	1.8	1.8	1.9	1.9	2.0
	C	297.2	267.5	237.8	208.1	178.4	148.7	118.9	89.2	59.5	29.7
1	G	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0
	C	296.3	266.7	237.1	207.4	177.8	148.2	118.5	88.9	59.3	29.6
0	G	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	C	295.5	265.9	236.4	206.8	177.3	147.7	118.2	88.6	59.1	29.5

and the expected numbers of survived guards and cumulative citizens are in Table 2.

If the number of available guards is less than four, we have to dispatch all guards regardless of the mission period. If the numbers are more than five, some guards have to be reserved during the initial phase of the entire periods for later stages. In the case that more than eight guards are available, the dispatching policy becomes clearer. All guards are not used up even in the final stage to avoid the victim by the detonation.

According to Table 2, the guards lose about 10 % of their own resources during the whole periods. On the other hand, the numbers of cumulative citizens decrease as many as 1 % in the same periods. As the guards must fight against the coming suicide bomber in our model, they suffer ten times as much damage as the cumulative citizens.

4.2 The effect of the number of arriving citizens Now we investigate the cases where the number of citizen $c(t)$ is shifted. In the cases of the number of citizen is 10, 50, 100 and 500 in every mission period, the optimal dispatching plans of guards are shown in Table 3. Other parameters are the same as the basic case in Section **4.1**.

In the cases of small number of the citizen, 10 and 50, we can see the reservation of some guards in every mission period. Dispatching more guards can cause more casualties of themselves. We should not dispatch so many guards to Arena for the small number of citizen.

Table 3. Optimal dispatching plans
(top left: $c(t) = 10$, top right: $c(t) = 50$, bottom left: $c(t) = 100$, bottom right: $c(t) = 500$)

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	4	5	5	5	5	5	5	5	5	5
9	5	5	5	5	5	5	5	5	5	5
8	4	4	4	4	4	4	5	5	5	5
7	4	4	4	4	4	4	4	4	5	5
6	4	4	4	4	4	4	4	4	4	5
5	4	4	4	4	4	4	4	4	4	5
4	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	8	8	8	8	8	8	8	8	8	8
9	7	7	7	7	7	8	8	8	8	8
8	5	5	6	7	7	7	7	8	8	8
7	5	5	5	5	5	7	7	7	7	7
6	4	4	5	5	5	6	6	6	6	6
5	4	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	10	10	10	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8	8
7	5	5	5	5	7	7	7	7	7	7
6	4	5	5	5	5	6	6	6	6	6
5	5	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	10	10	10	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8	8
7	5	7	7	7	7	7	7	7	7	7
6	5	5	5	5	6	6	6	6	6	6
5	5	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1

In the cases of 100 or more citizen, the best policy is dispatching all guards in hand. If the arena is crowded with many people, the explosive power of the bomb is blocked or weakened by the people near the bombing point. They serve as human shields to people far from the bombing point. All-guards-dispatching policy is the adaptive plan in such a situation. Only in the mid level of available guards $k = 7, 6$ and in earlier mission periods, the reserving policy is observed.

In the case of large number of the citizen, the blocking effect described above becomes more obvious. Also the vulnerability of each person in the Arena decreases as the size of the citizen increases. By those reasons, the best policy tends to be dispatching all guards.

4.3 The effect of the detection probability We investigate the relation between the optimal dispatching plan and the detection capability of each guard. By chaging the detection probability p from 0 to 1, we observe three types of dispatching rules. Two types of them are shown in Table 4.

For $p = 0 - 0.04$, the detection probability of each guard is too low for effective search. Dispatched guards may be killed for nothing by the bomber without detecting him. No-dispatching is reasonable choice to save the security resources and to lessen the casualties. If the guards have a little capability of detection of $p = 0.05 - 0.2$, all of them are dispatched to engage in the operation. They must gather all their capabilities to perform the best patrol. When the detection probability becomes bigger than 0.3, the optimal plan has a typical pattern as seen in Table 1; If the number of available guards are small, all of them must be dispatched to Arena. But if there are surplus guards, some of them are reserved for the later phase of mission periods. We can easily apply our model to the situation where the detection capability is different for each guard.

Table 4. Optimal dispatching plans (left: $p = 0 - 0.04$, right: $p = 0.05 - 0.2$)

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0

$k \setminus t$	10	9	8	7	6	5	4	3	2	1
10	10	10	10	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8	8
7	7	7	7	7	7	7	7	7	7	7
6	6	6	6	6	6	6	6	6	6	6
5	5	5	5	4	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1

5 Extension to multi-arena problem In the former sections, we have discussed about the single-arena problem. Now we extend our model to multi-arena situation. Sometimes the decision maker has the responsibility to dispatch guards to some Arenas simultaneously. For example, a manager in the patrol post wants to decide the number of dispatching guards to some different Arenas, from Arena 1 to Arena I. These Arenas have different properties in terms of the number of the arriving citizens or the size of them. The differences make the priority for attack/protection to suicide bombers/security guards. And as the result, the different attack scheme or guard policies are permissible for each arena. We will consider the two arena case at first.

5.1 Dispatch planning model to two arenas case As we have explained in Figure 1, any one of the five events can happen in every mission period both in Arena 1 and in Arena 2. Combining all possible events, there are 25 possible combinations of event for these Arenas. We assume that there are k available guards and x guards are dispatched to two Arenas from them. The x guards are divided into two groups, x_1 for Arena 1 and x_2 for Arena 2. In each Arena, a suicide bomber may arrive with probability λ_1, λ_2. In the t-th mission period, there are $c_1(t)$ and $c_2(t)$ citizens in Arena 1 and 2.

The optimal dispatching plan for Arena 1 and 2 is decided by the following recursive formula $FE_2[t,k]$. It calculates the maximized expectation of survived guards and citizens in Arena 1 and 2 from the beginning of the $t-$th period to the end of whole mission periods.

(9) $$FE_2[t,k] = \max_{\substack{0\le x\le k\\ x_1+x_2=x}} \sum_{j_1=1}^{5}\sum_{j_2=1}^{5} q^1_{j_1} q^2_{j_2} \Big[\Phi^1_{j_1}\Phi^2_{j_2}\big\{FE_2[t-1,k-D^s_{1,j_1}-D^s_{2,j_2}] + \sum_{i=1}^{2}(c_i(t)-D^c_{i,j_i})\big\}\Big].$$

The parameters j_1 and j_2 denote the possible event in Arena 1 and 2 respectively. The events are numbered from one to five as seen in Figure 1. Though it is not indicated explicitly, $q^i_{j_i}$, $\Phi^i_{j_i}$, $D^s_{j_i}$ and $D^c_{j_i}$ depend on x_i.

For the event j in Arena i, $q^i_{j_i}$ is the product of (not) arriving probability of a suicide bomber and (not) detection probability of guards. For instance, when a bomber comes to Arena i but the guards cannot detect him, it corresponds to the event 2 in Figure 1, then the parameter $q^i_{2_i}$ is $\lambda_i\delta(x_i)$. $\Phi^i_{j_i}$ is probabilities of the following events; the dispatched x_i guards decrease to y_i during the assault against the terrorist. So in cases that the battle does not happen, which correspond to the event 1 and 2 in Figure 1, the guards are not fatigued and the parameter is kept constant. On the other hand, when the battle takes place between the bomber and the guards, the residual y_i can take any one value between x_i and one. In these cases, $\Phi^i_{j_i}$ is the summation of probabilities. If all guards are defeated by a suicide bomber, it takes $p_{x_i}(1,0)$. Then multiply $\Phi^i_{j_i}$s by expectation of saved lives; the sum of guards and cumulative citizens up to the $(t-1)$-th period and citizens just in the t-th period. D^s_{i,j_i} and D^c_{i,j_i} are expected casualties on the guard and the citizen sides respectively. The initial condition is of course $FE_2(0,k)=k$ for any given k. The described parameters are summarized as follows.

$j_i = 1$: Not arrive

$q^i_1 = (1-\lambda_i),\ \Phi^i_1 = 1,\ D^s_{i,1} = 0,\ D^c_{i,1} = 0$.

$j_i = 2$: Not detect

$q^i_2 = \lambda_i\cdot\delta(x_i),\ \Phi^i_2 = 1,\ D^s_{i,2} = A^s_{i,0}(x_i, c_i(t)),\ D^c_{i,2} = A^c_{i,0}(x_i, c_i(t))$.

$j_i = 3$: Detect & Neutralize the bomber successfully

$q^i_3 = \lambda_i\cdot(1-\delta(x_i)),\ \Phi^i_3 = \sum_{y_i=1}^{x_i} p_{x_i}(0,y_i),\ D^s_{i,3} = x_i - y_i,\ D^c_{i,3} = 0$.

$j_i = 4$: Detect & Fail in Neutralization & $y_i \ge 1$

$q^i_4 = \lambda_i\cdot(1-\delta(x_i)),\ \Phi^i_4 = \sum_{y_i=1}^{x_i} p_{x_i}(1,y_i),\ D^s_{i,4} = x_i - y_i + A^s_{i,1}(y_i, c_i(t)),$
$D^c_{i,4} = A^c_{i,1}(y_i, c_i(t))$.

$j_i = 5$: Detect & Fail in Neutralization & $y_i = 0$

$q^i_5 = \lambda_i\cdot(1-\delta(x_i)),\ \Phi^i_5 = p_{x_i}(1,0),\ D^s_{i,5} = x_i,\ D^c_{i,5} = A^c_{i,0}(0, c_i(t))$.

5.2 Numerical examples and computational complexity We take two cases. We have $T=10$ mission periods and $k=20$ guards available. The numbers of citizens are assumed to be $c_1(t)=50$ and $c_2(t)=30$ persons, for every period t. More visitors come to Arena 1 than to Arena 2. Other parameters are set the same as in the single arena case in Section 4.1. The optimal number of guards for Arena 1, x^*_1, and for Arena 2, x^*_2, are summarized in Table 5.

Because Arena 1 is more valuable than Arena 2, more guards are dispatched to Arena 1 in almost all mission periods. However, the difference is slight. The dispatching policies are almost the same as in the single arena case. Even though the available guards are enough, at most 8 or 7 guards are dispatched to avoid damage to the guards themselves. In the cases of $k = 15, 13$ and 12, there are slight changes in optimal numbers of dispatching guards. When the available guards are deficient and k varies from 11 to 3, all guards are divided into almost same numbers for each Arena. In the case of $k = 2$, the optimal dispatching plan is $(2, 0)$ in every mission period.

Table 5. The optimal dispatching plan (x_1^*, x_2^*)

$k\backslash t$	10	9	8	7	6	5	4	3	2	1
20	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)
19	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)
18	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)
17	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)
16	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)
15	(8,6)	(8,6)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)	(8,7)
14	(8,6)	(8,6)	(8,6)	(8,6)	(8,6)	(8,6)	(8,6)	(8,6)	(8,6)	(8,6)
13	(8,5)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)
12	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(6,6)
11	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)
10	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)	(5,5)
9	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)
8	(4,4)	(4,4)	(4,4)	(4,4)	(4,4)	(4,4)	(4,4)	(4,4)	(4,4)	(4,4)
7	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)
6	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)
5	(3,2)	(3,2)	(3,2)	(3,2)	(3,2)	(3,2)	(3,2)	(3,2)	(3,2)	(3,2)
4	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)	(2,2)
3	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
2	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)
1	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)

Table 6. The optimal dispatching plan (x_1^*, x_2^*) ($\lambda_1 = 1/30, \lambda_2 = 1/300$)

$k\backslash t$	10	9	8	7	6	5	4	3	2	1
20	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
19	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
18	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
17	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
16	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
15	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
14	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)	(7,7)
13	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)	(7,6)
12	(6,6)	(6,6)	(6,6)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)	(7,5)
11	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)	(6,5)
10	(6,4)	(6,4)	(6,4)	(6,4)	(6,4)	(6,4)	(6,4)	(6,4)	(6,4)	(6,4)
9	(5,4)	(5,4)	(5,4)	(5,4)	(5,4)	(6,3)	(6,3)	(6,3)	(6,3)	(6,3)
8	(5,3)	(5,3)	(5,3)	(5,3)	(5,3)	(5,3)	(5,3)	(5,3)	(5,3)	(5,3)
7	(4,3)	(4,3)	(5,2)	(5,2)	(5,2)	(5,2)	(5,2)	(5,2)	(5,2)	(5,2)
6	(4,2)	(4,2)	(4,2)	(4,2)	(4,2)	(4,2)	(4,2)	(4,2)	(4,2)	(5,1)
5	(4,1)	(4,1)	(4,1)	(4,1)	(4,1)	(4,1)	(4,1)	(5,0)	(5,0)	(5,0)
4	(4,0)	(4,0)	(4,0)	(4,0)	(4,0)	(4,0)	(4,0)	(4,0)	(4,0)	(4,0)
3	(3,0)	(3,0)	(3,0)	(3,0)	(3,0)	(3,0)	(3,0)	(3,0)	(3,0)	(3,0)
2	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)	(2,0)
1	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)

We also calculated another case that the arriving probabilities of a suicide bomber to each arena, λ_1 and λ_2, are different. We assumed that Arena 1 is more dangerous ($\lambda_1 = 1/30$) than Arena 2 ($\lambda_2 = 1/300$) for the calculation. To consider the relation between λ_i and optimal number of dispatching guards, $c_1(t)$ and $c_2(t)$ are set to be the same 30 for any mission period t. Other parameters are also set the same 30 as the former examples. The optimal dispatching plan for the case is Table 6.

The properties of the plan are almost same as Table 5. If they have sufficient guards to be dispatched, the optimal dispatching policy is $(7,7)$ to avoid the victim. When the available guards are $k = 12, 9, 7, 6$ and 5, shifts of one guard from Arena 2 to more dangerous Arena 1 are observed. And in more severe conditions that $k \leq 4$, all the guards must be dispatched to Arena 1 to save more lives of the citizens and the guards.

By those numerical results, we showed that we could easily extend the single arena formulation to two arenas case. We conclude that our method is useful for deciding the delicate dispatch planning even in two arena case.

We also investigate the relations between the input size of the problem and the computational time. At first, we can easily check that the computational time is proportional to T. To calculate values for the t-th period, $FE[t,k]$, we must use $FE[t-1,0], \cdots, FE[t-1,k]$, so the whole computational time increases by input size T linearly. The linearity is also valid for $FE_2[t,k]$.

On the other hand, K dependency of the computation time is more complicated. The dependencies are different in single and two arenas cases. In the single arena case, when we calculate by Eq.(7), we have to shift x from 0 to k. Moreover, when the assault battle is taken place, the parameter y must be changed from 1 to x to calculate the expectation. In each of these two cases, we have to calculate x cases. As the result, we must consider $\sum_{x=1}^{k}(2x+3) = k^2 + 4k$ states when $k \neq 0$ and 2 states in $k = 0$. When no guard is dispatched, not five but just two events, a terrorist does not appear or he can explode his bomb freely at the center of Arena, would happen. By those considerations, the computational complexity is proportional to $4k^2 + 4k + 2$ or $O(k^2)$ in the rough. So the complexity increases by $O(TK^2)$ in the single arena case. As for the two arenas case, the complexity becomes $O(TK^4)$.

5.3 Dispatch planning model to multiple arenas case Now we investigate the problem with more than three arenas. From now on, we consider *I-arena problem.* By the analogy of Eq.(9), the recursive formula of *I-arena problem* is written as follows.

$$(10)\quad FE_I[t,k] = \max_{\substack{0 \leq x \leq k \\ x_1+\cdots+x_I=x}} \sum_{j_1=1}^{5} \cdots \sum_{j_I=1}^{5} q_{j_1}^1 \cdots q_{j_I}^I \Big[\Phi_{j_1}^1 \cdots \Phi_{j_I}^I \big\{FE_I[t-1, k-\sum_{i=1}^{I} D_{i,j_i}^s] + \sum_{i=1}^{I}(c_i(t) - D_{i,j_i}^c)\big\}\Big].$$

The complexity also increases linearly in proportion to T. K dependency, however, seems to be very complicated and hard to analyze even three arenas case.

The analytical process is as follows. First, to distribute x guards for I arenas, we must divide x into I blocks. We will denote each of them by $x_1, x_2, \cdots, x_I$. Any of them, but not all at once, is allowed to be zero. All dividing patterns $\{(x_1, x_2, \cdots, x_I)\}$ are the number of the repeated combination and it becomes ${}_IH_x = {}_{I+x-1}C_x$.

In each of the distributed pattern element $(x_1, x_2, \cdots, x_I)$, we must distinguish two different possibilities; the component x_i equals zero or not. If x_i equals zero, it means that no guard is dispatched to Arena i. Then, only two events, a terrorist does not show up or come and bomb at the center, would happen. We assume that in z out of I arenas, x_i equals zero. On the other hand, if x_i is not zero, we must consider five possible events and the number of the total possible states becomes $2x_i + 3$ as we have considered in the previous section. As the result, the total number of the combined possible states becomes $2^z \cdot \prod_{i=1}^{I-z}(2x_i + 3)$ for each of the element. Note that in this estimation, x_is are sorted and renumbered; five possible events may happen in the first $I - z$ arenas and no guard is dispatched to the last z arenas. And in the former arenas, some x_is may take the same value. For those x_is, we define the set $U = \{x_i | x_i = \alpha\}, V = \{x_i | x_i = \beta\}$ and so on. When we calculate the number of the whole possible states, we must multiply $I!/(|z|!|U|!|V|! \cdots)$ by the value for an element and then shift x from 0 to k. Through those procedures, we can get the number of total states but it is a tough work even in small I arenas case.

6 Summary In this paper, we proposed a planning model of dispatching security guards to a hazardous arena. The proposed method produces reasonable and acceptable plans to save the lives of citizen by the limited number of security staffs. In accordance with the detection probability of each guard, the optimal dispatching policy changes; if the probability is very low, none of them must be dispatched to save the security resources. If it increases a little, all guards must be dispatched. When the guards have relatively high capability, not all, but just some of them must be dispatched. The model gives us a quantitative analysis about the best timing or the optimal number of dispatching guards.

The single arena model is easily extendable to multi-arena cases. The solution for two-arena case also has the same properties of the single-arena one. For the case of 3 or more arenas, counting up of whole combination of possible states is hard and the number will not be expressed in a simple formula. To solve more than three arenas problem efficiently, we should seek for heuristic algorithms.

To improve our model, we have the following future works. Our model was built on many simplified assumptions. The shape of the arena was circular, as assumed by Kress [7], the detection probabilities of each guard are set the same and they are a constant regardless of the situations or personal abilities. They search for suspicious guys independently. But in the real operations, the shape of the arena is rectangle in most cases. They will patrol by buddy. We have to modify our model to fit for the real world. Also we used many probabilities in the model. Those probabilities, such as the urge of the suicide bomber S or the detection probability p, must be evaluated from some psychological knowledge or training data. As the scientific study about counter terrorism has just begun now, we have to prepare many input data as well as the models. Also we have to consider some real operational restrictions. For the optimal number of dispatching guards, we only take into account the number. However we have to consider other conditions such as watch schedule and/or the cost of guards. In multi-arena cases, we have to investigate the problem when there is some difference in the patrol periods among arenas.

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Appendix A. The expectation of the casualties when the bomb explodes during the battle We will explain the computational procedure of the expected casualties when a suicide bomber detonates his bomb during the battle. We referred to the idea in [7]. The suicide bomber can detonate his bomb at any point in the arena. If nobody detects him, he can go to the center of the circular arena and maximize the damage. Or if he is detected and the battle begins, he may push the button there. In that case, the detonation would happen not at the center in general. But as the possible occupied areas of persons are decided previously, we first compute the casualties when the bomb explodes at each area in the arena and then average those expected casualties.

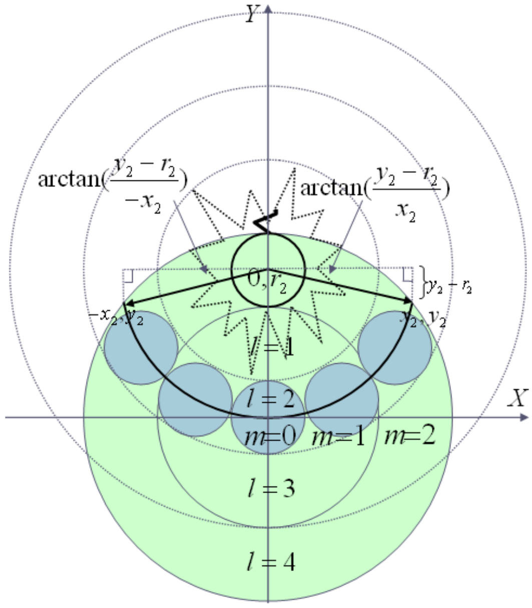

Figure 3 When the bomb detonates in m=2$_{nd}$, there are 6 possible seats in l=2$_{nd}$

As seen in Figure 3, we set the center of the arena as the origin of the coordinates, $(0,0)$ and the circular rings of the arena are indexed by m. Then we assume that the bomb explodes at $(0, r_2)$ and the bombing ring is $l = 0$. By the symmetry of the arena, we can choose the exploded point arbitrarily. As the width of each ring of the arena and the bombing ring are the same, the rings of the arena, ms, and the bombing rings, ls, overlap wholly with each other.

At first, we will estimate the expected number of people in the l th bombing ring area when the bomb explode at the m th arena, $\mu_{m,l}$. Before doing that, we calculate the maximum possible number of people in part of the ring, $a_{m,l}$. As a person occupies 2 θ_l on the lth ring,

$$\begin{aligned} a_{m,l} &= [180^\circ + 2\arctan(\frac{y_l - r_m}{x_l})]/2\theta_l \\ &= [180^\circ + 2\arctan(\frac{y_l - r_m}{x_l})]/2\arcsin(\frac{1}{2l}) \ , \end{aligned} \tag{11}$$

where $r_m = 0.5m$ [meter]. As the diameter of the human body is assumed 0.5 [meter] in [7], the radius of the m th ring, r_m, is expressed as above. (x_l, y_l) is the intersection of the following two circle $(x_l > 0)$. M is the diameter, or the number of the rings, of the circular arena.

$$\begin{array}{lcl} \text{Circular arena} & : & x^2 + y^2 = (0.5 \cdot M + 0.25)^2. \\ \text{Circular area from the explosion} & : & x^2 + (y - r_m)^2 = (0.5l)^2. \end{array}$$

In each of the lth ring, people can overlap a little, so we will round up $h_{m,l} = \lceil a_{m,l} \rceil$. When there are C persons in the arena uniformly, the expected number of people in the lth region, $\mu_{m,l}$ is

$$\mu_{m,l} = \frac{h_{m,l}}{H(M+m)} C \ , \tag{12}$$

where $H(M+m)$ is the maximum possible people in the arena.

$$H(M+m) = \sum_{l=1}^{M+m} h_{m,l}. \tag{13}$$

As an illustrative example, we assume that the diameter of the arena is 2 meter ($M = 2$). When the explosion happens in $m = 2_{nd}$ ring, the maximum seats in $l = 2_{nd}$ ring, $a_{2,2}$ is

$$a_{2,2} \quad = \quad [180^\circ + 2\arctan(\frac{0.78 - 0.5 \cdot 2}{0.98})]/2\arcsin(\frac{1}{2 \cdot 2}) = 5.3, \tag{14}$$

so $h_{2,2} = 6$ persons. Then the maximum possible persons in the arena are $H(2+2) = \sum_{l=1}^{2+2} h_{2,l} = \lceil 3.6 \rceil + \lceil 5.3 \rceil + \lceil 5.8 \rceil + \lceil 4.3 \rceil = 21$ persons. If there are $C = 7$ persons in the arena, $\mu_{2,2} = 6/21 \times 7 = 2.0$ persons.

When there are y guards and $c(t)$ citizens and the bomber detonates in the m th ring, the total casualties are computed by the analogy of Eq.(4).

$$Acas_m(M, y + c(t)) = \sum_{l=1}^{M+m} \mu_{m,l} \cdot \gamma(l) \cdot P_H(l). \tag{15}$$

If there are 3 guards and 47 citizens in the arena, whose diameter is 5 meter and the detonation occur at the $m = 3_{rd}$ ring, the expectation is computed as follows.

$$\begin{aligned} Acas_3(5, 3+47) &= \sum_{l=1}^{5+3} \mu_{3,l} \cdot \gamma(l) \cdot P_H(l) \\ &= 3.18 + 3.12 + 1.62 + 0.76 + 0.34 + 0.14 + 0.05 + 0.01 = 9.23 \text{ (persons)}. \end{aligned}$$

As the detonation may happen at any seat in the arena, the expected casualties during the battle, $Acas(M, y+c(t))$, are calculated by taking an weighted average of $Acas_m(M, y+c(t))$ by the m th seats in the arena.

$$Acas(M, y + c(t)) = \left[\sum_{m=0}^{M} Acas_m(M, y + c(t)) \cdot h(m) \right] / \sum_{m=0}^{M} h(m). \tag{16}$$

In this example, the value becomes

$$\begin{aligned} Acas(5, 3+47) &= \left[\sum_{m=0}^{5} Acas_m(5, 50) \cdot h(m) \right] / \sum_{m=0}^{5} h(m) \\ &= [10.46 \cdot 1 + 10.28 \cdot 6 + 10.04 \cdot 13 + 9.23 \cdot 19 + 7.84 \cdot 26 + 5.69 \cdot 32] /97 \\ &= 7.88 \text{ (persons)}. \end{aligned}$$

As the diameter of the arena M is assumed to be a constant throughout each of the numerical examples, we omitted the parameter from the notation and denote the casualties in simpler form like $A_0^c(x, c(t)), A_1^s(y, c(t))$ as defined in Eq.(4). In this example, $A_1^s(3, 47)$ and $A_1^c(3, 47)$ are calculated by Eqs.(5) and (6): $A_1^s(3, 47) = 7.88 \times 3/50 = 0.47$ (persons) and $A_1^c(3, 47) = 7.88 \times 47/50 = 7.41$ (persons).

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