LINE MATCHING FROM STEREO TREE IMAGES USING NEURAL NETWORK

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Abstract. In this paper, the matching of center lines from stereo tree images is considered, and formulated as a combinatorial optimization problem. Marr and Poggio showed that the Hopfield network, a kind of artificial neural network model, can be applied to reconstruct 3D structure from random dot stereogram. Their method consists of the following five steps; 1.form matching candidates, 2.allocate candidates on units of network, 3.determine connection weights among units from the characteristics of images, 4.converge to a stable state and 5.reconstruct 3D structure from the state of network. This paper proposes a new formation of connection weights in the stereo line matching algorithm.

1 Introduction The main purpose of the line matching from stereo tree images is to recover depth information of an object in a scene using a pair of stereo images. There are several approaches for solving the matching problem. One is to match every point in the left image with that of the right image. Another is to extract distinct features from each image and using some constraints the corresponding features are determined.

In this paper, matching every point from stereo images by using the neural networks is considered. It can be formulated as minimization of a energy function where all the constraints can explicitly be included. Minimization of the energy function can then be performed by a Hopfield network or by a stochastic optimization such as a simulated annealing.

2 Hopfield network The stereo line matching problem can be formulated as an optimization task where a energy function representing the constraints on the stereo solution is minimized. The minimization problem can be mapped onto a Hopfield network with the interconnection weights representing the constraints imposed by the matching problem. When the network is at its stable state the energy function is said to be at its local minimum. The basic principle of the neural networks is to make a cooperative decision based on the simultaneous input of a whole community of units in which each unit receives information from and gives information to every other unit. This information is used by each unit to force the network to converge to a stable state in order to make a decision.

A two-dimensional binary Hopfield network is used to find the correspondence between the interesting points in the left and right images. The state of each unit in the network represents a possible match.

The energy function for a two-dimensional binary Hopfield network is given by

$$E(v_1, v_2, ..., v_N) = -\frac{1}{2}\sum_{i,j=1}^{N} w_{ij}v_iv_j - \sum_{i=1}^{N}\theta_i v_i$$

where N is the total number of units, v_i and v_j represent the binary state of i-th and j-th units, respectively, which can be either 1(active) or 0(inactive), w_{ij} is the connection weight between the two units, the self-feedback to each unit is $w_{ii} = 0$, and θ_i is the initial input to each unit. A change in the state of i-th unit by Δv_i will cause an energy change ΔE:

$$\Delta E = -(\sum_{j=1}^{N} w_{ij}v_j + \theta_i)\Delta v_i.$$

The above equation describes the dynamics of the network which was shown by Hopfield to be always negative with a updating rule

$$\begin{array}{c} v_i \rightarrow 0 \;\; if(\sum_{j=1}^{N} w_{ij}v_j + \theta_i) > 0 \\ v_i \rightarrow 1 \;\; if(\sum_{j=1}^{N} w_{ij}v_j + \theta_i) < 0 \\ no\ change\ if(\sum_{j=1}^{N} w_{ij}v_j + \theta_i) = 0. \end{array}$$

3 The Energy Minimization

3.1 Definitions Given $M \times N$ stereo binary images, let $\boldsymbol{P}$ denote the set of three dimensional vector:

$$\boldsymbol{P} = \{(x, y, z) \in \boldsymbol{N}^3 | x, y \in [0, M), z \in [0, N)\}$$

with:

$$p_i = (x_i, y_i, z_i) \in \boldsymbol{P}.$$

Let $f(x, y), g(y, z)$ denote a value for a pixel

$$f(x, z), g(y, z) \in \{0, 1\}(0 \leq x, y < M, 0 \leq z < N).$$

The set of correspondence candidates is defined as follows:

$$P_m = \{(x, y, z) \in \boldsymbol{P} | f(x, z) = 1 \wedge g(y, z) = 1\}.$$

Let Q denote the number of elements in P_m,

$$Q = \sum_{z=0}^{N-1} q_x^z q_y^z$$

where q_x^z is the number of elements in X-axis direction and q_y^z is the number of elements in Y-axis direction on $Z = z$ plane.

The set of uniqueness P_u, a subset of P_m, is defined as follows.

$\forall p_i \in P_u, \exists p_j \notin P_u, z_i = z_j \wedge$

$$\begin{cases} x_i = x_j \wedge y_i \neq y_j & , q_x^{z_i} > q_y^{z_i} \\ x_i \neq x_j \wedge y_i = y_j & , q_x^{z_i} < q_y^{z_i} \\ (x_i = x_j \wedge y_i \neq y_j) \vee & \\ \quad (x_i \neq x_j \wedge y_i = y_j) & , q_x^{z_i} = q_y^{z_i}. \end{cases}$$

Similarly, the set of continuity P_c, a subset of P_m, is defined as follows.

$$
\begin{aligned}
&\forall p_i \in P_c, \exists p_j \in P_c,\\
&\qquad |x_i - x_j|, |y_i - y_j| \le 1, |z_i - z_j| = 1.
\end{aligned}
$$

Let $\boldsymbol{P}_{uc}$ denote the set of uniqueness and continuity.

The set of optimal candidate P^*, a subset of P_m, is defined as follows.

(a) $P^* \in \boldsymbol{P}_{uc}$

(b)

$$
\begin{aligned}
&\forall p_i \in P^*, \exists p_j \in \boldsymbol{P}_{uc},\\
&\qquad |x_i - x_j|, |y_i - y_j| \le 1, |z_i - z_j| = 1\\
&\qquad \Rightarrow p_j \in P^*
\end{aligned}
$$

3.2 Energy function Assume $\boldsymbol{P}^* \neq \phi$, Q dimensional energy function is defined as follows.

$$
\begin{aligned}
&E(\boldsymbol{v}) = E(v_{x_1y_1z_1}, v_{x_2y_2z_2}, ..., v_{x_Qy_Qz_Q}) =\\
&\quad -\frac{1}{2}\sum_{i,j=1}^{Q} w_{x_iy_iz_i,x_jy_jz_j} v_{x_iy_iz_i} v_{x_jy_jz_j}\\
&\quad -\sum_{i=1}^{Q} \theta_{x_iy_iz_i} v_{x_iy_iz_i}
\end{aligned}
$$

where $\boldsymbol{v} = \{v_{x_1y_1z_1}, v_{x_2y_2z_2}, ..., v_{x_Qy_Qz_Q}\} \in \boldsymbol{V} = \{-1, 1\}^Q$.

For each $p_i, p_j \in P_m$,

(a) If p_i and p_j are contained in the same set of uniqueness, then

$$w_{x_iy_iz_i,x_jy_jz_j} = -\frac{1}{2}.$$

(b) If p_i and p_j are contained in the same set of continuity, and

$$
\begin{aligned}
&\exists p_k \notin P_m, p_j \neq p_k,\\
&\qquad |x_i - x_k|, |y_i - y_k| \le 1, |z_i - z_k| = 1,
\end{aligned}
$$

then

$$w_{x_iy_iz_i,x_jy_jz_j} = 1.$$

(c) Otherwise, $w_{x_iy_iz_i,x_jy_jz_j} = 0$.

The initial input is defined as follows.

$$
\theta_{x_iy_iz_i} = \begin{cases} 1 - \frac{q_y^{z_i}}{2} & , q_x^{z_i} > q_y^{z_i}\\ 1 - \frac{q_x^{z_i}}{2} & , q_x^{z_i} < q_y^{z_i}\\ 2 - \frac{q_x^{z_i}}{2} - \frac{q_y^{z_i}}{2} & , q_x^{z_i} = q_y^{z_i}. \end{cases}
$$

For each set of optimal candidate $P^* \in \boldsymbol{P}^*$, Q dimensional vector $\boldsymbol{v}^* \in \boldsymbol{V}$ is defined as follows.

$$v^*_{x_i,y_i,z_i} = \begin{cases} 1, & p_i \in P^* \\ -1, & p_i \notin P^* \end{cases} \quad (1 \le i \le Q).$$

Then

$$\forall \boldsymbol{v} \in \boldsymbol{V}, E(\boldsymbol{v}) \ge E(\boldsymbol{v}^*)$$

holds. Therefore $\boldsymbol{v}^*$ is the globally optimum solution. And let C be the number of pairs subject to the condition (b). Then

$$\inf E(\boldsymbol{v}) = E(\boldsymbol{v}^*) = -C$$

holds.

Proof:

The energy function can be divided into the term E_u^z related to units on same X-Y plane and the term E_c^z related to units on parallel X-Y planes as follows.

$$\begin{aligned} E(\boldsymbol{v}) &= -\frac{1}{2} \sum_{x_i y_i z_i, x_j y_j z_j} w_{x_i y_i z_i, x_j y_j z_j} v_{x_i y_i z_i} v_{x_j y_j z_j} \\ &\quad - \sum_i \theta_{x_i y_i z_i} v_{x_i y_i z_i} \\ &= \sum_{z=0}^{N-2} E_c^z(\boldsymbol{v}) + \sum_{z=0}^{N-1} E_u^z(\boldsymbol{v}) \end{aligned}$$

where

$$E_c^z(\boldsymbol{v}) = - \sum_{x_i, x_j, y_i, y_j} w_{x_i y_i z, x_j y_j z+1} v_{x_i y_i z} v_{x_j y_j z+1}$$

$$\begin{aligned} E_u^z(\boldsymbol{v}) &= -\frac{1}{2} \sum_{x_i, x_j, y_i, y_j} w_{x_i y_i z, x_j y_j z} v_{x_i y_i z} v_{x_j y_j z} \\ &\quad - \sum_{x_i, y_i} \theta_{x_i y_i z} v_{x_i y_i z}. \end{aligned} \tag{1}$$

First we consider the weighted connections on X-Y plane such as $Z = z$.

1. Case 1:$q_x^z > q_y^z$

   Let the energy function be as follows, then output vector such that a single unit in Y-axis direction fires is the globally optimum solution.

$$\sum_{x_i}(\sum_{y_i}\frac{v_{x_iy_iz}+1}{2}-1)^2$$

It can be expanded as follows.

$$\begin{aligned}
&= \sum_{x_i}(\frac{1}{4}\sum_{y_i}v^2_{x_iy_iz}+\frac{q_y^{z2}}{4}+1+\frac{1}{2}\sum_{y_i\neq y_j}v_{x_iy_iz}v_{x_jy_jz} \\
&\quad +\frac{1}{2}q_y^z\sum_{y_i}v_{x_iy_iz}-\sum_{y_i}v_{x_iy_iz}-q_y^z) \\
&= \sum_{x_i}\{\frac{q_y^{z2}}{4}-\frac{3}{4}q_y^z+1+\frac{1}{2}\sum_{y_i\neq y_j}v_{x_iy_iz}v_{x_jy_jz} \\
&\quad +(\frac{1}{2}q_y^z-1)\sum_{y_i}v_{x_iy_iz}\} \\
&= \frac{q_x^zq_y^{z2}}{4}-\frac{3}{4}q_x^zq_y^z+q_x^z+\frac{1}{2}\sum_{x_i}\sum_{y_i\neq y_j}v_{x_iy_iz}v_{x_jy_jz} \\
&\quad -(1-\frac{1}{2}q_y^z)\sum_{x_i}\sum_{y_i}v_{x_iy_iz}
\end{aligned} \tag{2}$$

2. Case 2:$q_x^z < q_y^z$

Let the energy function be as follows.

$$\sum_{y_i}(\sum_{x_i}\frac{v_{x_iy_iz}+1}{2}-1)^2.$$

Similarly, it can be expanded as follows.

$$\begin{aligned}
&\frac{q_y^zq_x^{z2}}{4}-\frac{3}{4}q_x^zq_y^z+q_y^z+\frac{1}{2}\sum_{y_i}\sum_{x_i\neq x_j}v_{x_iy_iz}v_{x_jy_jz} \\
&\qquad -(1-\frac{1}{2}q_x^z)\sum_{x_i}\sum_{y_i}v_{x_iy_iz}.
\end{aligned} \tag{3}$$

3. Case 3:$q_x^z = q_y^z$

Let the energy function be as follows, then output vector such that a single unit in X-axis direction and a single unit in Y-axis direction fire is the globally optimum solution.

$$\sum_{y}(\sum_{x}\frac{v_{xyz}+1}{2}-1)^2+\sum_{x}(\sum_{y}\frac{v_{xyz}+1}{2}-1)^2$$

$$= \frac{1}{2} q_x^z q_y^z + q_x^z (\frac{q_y^z}{2} - 1)^2 + q_y^z (\frac{q_x^z}{2} - 1)^2 \tag{4}$$
$$+ \frac{1}{2} \sum_y \sum_{x_i \neq x_j} v_{x_i y z} v_{x_j y z} + \frac{1}{2} \sum_x \sum_{y_i \neq y_j} v_{x y_i z} v_{x y_j z}$$
$$- (2 - \frac{q_x^z}{2} - \frac{q_y^z}{2}) \sum_{x,y} v_{xyz}.$$

Comparing (1) with (2),(3),(4),

$$\begin{cases} x_i = x_j \wedge y_i \neq y_j & , q_x^z > q_y^z \\ x_i \neq x_j \wedge y_i = y_j & , q_x^z < q_y^z \\ (x_i = x_j \wedge y_i \neq y_j) \vee & \\ \quad (x_i \neq x_j \wedge y_i = y_j) & , q_x^z = q_y^z \end{cases}$$

then

$$w_{x_i y_i z, x_j y_j z} = -\frac{1}{2}$$

$$\theta_{x_i y_i z} = \begin{cases} 1 - \frac{q_y^z}{2} & , q_x^z > q_y^z \\ 1 - \frac{q_x^z}{2} & , q_x^z < q_y^z \\ 2 - \frac{q_x^z}{2} - \frac{q_y^z}{2} & , q_x^z = q_y^z. \end{cases}$$

Therefore,

$$\forall z \in [0, N) \cap \boldsymbol{N}, \forall \boldsymbol{v} \in \boldsymbol{V}, E_u^z(\boldsymbol{v}) \geq E_u^z(\boldsymbol{v}^*) \tag{5}$$

holds.

Secondly, we consider the weighted connection between units on parallel X-Y plane such as $Z = z, z + 1 (0 \leq z < N - 1)$.

$$E_c^z = - \sum_{x_i, y_i, x_j, y_j} w_{x_i y_i z, x_j y_j z+1} v_{x_i y_i z} v_{x_j y_j z+1}.$$

In case of $v_i = 1$, p_i and p_j are connected continuously in the term such as $w_{ij} = 1$. Because of $p_i \in \boldsymbol{P}^*$, a single point such that connects P_i continuously exists, hence it is identical to p_j. From the definition of $\boldsymbol{P}^*$, $p_j \in \boldsymbol{P}^*$, we get $p_j \in \boldsymbol{P}^*$ and $v_j = 1$.

Similarly we have

$$v_i = 1 \Leftrightarrow v_j = 1, v_i = -1 \Leftrightarrow v_j = -1.$$

Generally for 2 binary variables $x_i \in \{-1, 1\} (i = 1, 2)$,

$$f(x_1, x_2) = -a x_1 x_2 (a > 0) \tag{6}$$

is minimized at $(x_1, x_2) = (-1, -1), (1, 1)$.

Since the element in $\boldsymbol{P}^*$ minimizes the terms of E_c^z, it minimizes E_c^z which is the sum of each term.

Therefore

$$\forall z \in [0, N) \cap \boldsymbol{N}, \forall \boldsymbol{v} \in \boldsymbol{V}, E_c^z(\boldsymbol{v}) \geq E_c^z(\boldsymbol{v}^*) \tag{7}$$

holds.

Thirdly, by adding Eq.(5) to Eq.(7), we obtain

$$\forall \boldsymbol{v} \in \boldsymbol{V}, \forall \boldsymbol{v}^* \in \boldsymbol{V}^*, E(\boldsymbol{v}) \geq E(\boldsymbol{v}^*).$$

And since $E_u^z(\boldsymbol{v}^*) = 0$ for any z, we get

$$\inf E = E(\boldsymbol{v}^*) = -C.$$

∎

4 Conclusion In this paper, a method to solve the stereo correspondence problem based on Hopfield network has been presented. Two given stereo tree images are assumed to satisfy the constraints of uniqueness and continuity. The formation to define an energy function which global minimum corresponds to the optimal combination of correspondence candidates is proposed.

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