ON IMPLICATIVE HYPER K-IDEALS OF HYPER K-ALGEBRAS

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ABSTRACT. The notion of s-implicative hyper K-ideals is introduced, and then some of related properties are investigated.

1. INTRODUCTION

The hyper structure theory (called also multialgebra) was introduced in 1934 by Marty [7] at the 8th congress of Scandinavian Mathematiciens. Hyper structures have many applications to several sectors of both pure and applied sciences. Recently, Jun et al. [6] and Borzoei et al. [3] applied the hyper structure to BCK/BCI-algebras. In [2], Borumand Saeid et al. defined the notion of (weak) implicative hyper K-ideals in hyper K-algberas, and classified the implicative hyper K-ideals of order 3. The aim of this paper is to define the notion of s-implicative hyper K-ideals, and then we investigate some related properties. we prove that implicative hyper K-ideals and s-implicative hyper K-ideals coincide. we give a condition for a (strong) hyper K-ideal to be an (s-)implicative hyper K-ideal. We also introduced the notion of implicative hyper K-algebras and we give a condition for a hyper K-algebra to be implicative.

2. PRELIMINARIES

We include some elementary aspects of hyper K-algebras that are necessary for this paper, and for more details we refer to [3] and [8]. Let H be a non-empty set endowed with a hyper operation "$\circ$", that is, $\circ$ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$. For two subsets A and B of H, denote by $A \circ B$ the set $\bigcup_{a\in A, b\in B} a \circ b$.

By a *hyper I-algebra* we mean a non-empty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:

(H1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(H2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(H3) $x < x$,
(H4) $x < y$ and $y < x$ imply $x = y$

for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a < b$. If a hyper I-algebra $(H, \circ, 0)$ satisfies an additional condition:

(H5) $0 < x$ for all $x \in H$,

then $(H, \circ, 0)$ is called a *hyper K-algebra* (see [3]).

In a hyper I-algebra H, the following hold (see [3, Proposition 3.4]):

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(p1) $(A \circ B) \circ C = (A \circ C) \circ B$,
(p2) $A \circ B < C \Leftrightarrow A \circ C < B$,
(p3) $A \subseteq B$ implies $A < B$

for all nonempty subsets A, B and C of H.

In a hyper K-algebra H, the following hold (see [3, Proposition 3.6]):

(p4) $x \in x \circ 0$ for all $x \in H$.

Definition 2.1. [3] A nonempty subset I of a hyper K-algebra H is called a *weak hyper K-ideal* of H if it satisfies the following conditions:

(I1) $0 \in I$,
(I2) $\forall x, y \in H,\ x \circ y \subseteq I, y \in I \Rightarrow x \in I$.

Definition 2.2. [3] A nonempty subset I of a hyper K-algebra H is called a *hyper K-ideal* of H if it satisfies (I1) and

(I3) $\forall x, y \in H,\ x \circ y < I, y \in I \Rightarrow x \in I$.

Note that every hyper K-ideal is a weak hyper K-ideal, but the converse is not true (see [3, Proposition 4.6 and Example 4.7]).

Definition 2.3. [5] A nonempty subset I of a hyper K-algebra H is called a *strong hyper K-ideal* of H if it satisfies (I1) and

(I4) $\forall x, y \in H,\ (x \circ y) \cap I \neq \emptyset, y \in I, \Rightarrow x \in I$.

Note that every strong hyper K-ideal is both a weak hyper K-ideal and a hyper K-ideal. (see [5, Theorem 3.6])

Definition 2.4. [4] A nonempty subset I of a hyper K-algebra H is called a *positive implicative hyper K-ideal of type-1* and *type-2*, respectively, if it satisfies (I1) and

(I5) $\forall x, y, z \in H,\ (x \circ y) \circ z \subseteq I, y \circ z \subseteq I \Rightarrow x \circ z \subseteq I$,
(I6) $\forall x, y, z \in H,\ (x \circ y) \circ z < I, y \circ z \subseteq I \Rightarrow x \circ z \subseteq I$,

respectively.

Definition 2.5. [4] Let I be a nonempty subset of a hyper K-algebra H. Then we say that I satisfies the *additive condition* if, for all $x, y \in H$, $x < y$ and $y \in I$ imply that $x \in I$.

Theorem 2.6. [3, Lemma 4.9] *Every hyper K-ideal of a hyper K-algebra satisfies the additive condition.*

Definition 2.7. [1] A hyper K-ideal I is said to be *reflexive* if $x \circ x \subseteq I$ for all $x \in H$.

Definition 2.8. [8] Let H be a hyper K-algebra. If there exists an element $u \in H$ such that $x < u$ for all $x \in H$, then H is called a *bounded hyper K-algebra* and u is called the *unit* of H.

3. Implicative hyper K-ideals

We first consider the following implication in a hyper K-algebra H for every $x, y, z \in H$.

(q1) $y < z \Rightarrow x \circ z < x \circ y$.

In general, the implication (q1) does not hold in a hyper K-algebra as shown in the following example.

Example 3.1. Let $H = \{0, a, b, c\}$ be a set and consider the following table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a, b\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$	$\{0, b\}$
c	$\{c\}$	$\{0, b\}$	$\{c\}$	$\{0, c\}$

Then $(H, \circ, 0)$ is a hyper K-algebra. Note that $a < b$ because $0 \in a \circ b$, and $c \circ b \not< c \circ a$ since $0 \notin (c \circ b) \circ (c \circ a)$.

However we can find hyper K-algebras in which the implication (q1) holds. For example, the implication (q1) is true in a hyper K-algebra of order 3 or a bounded hyper K-algebra of order 4 as mentioned in the following propositions.

Lemma 3.2. *In a hyper K-algebra H, we have $x \circ y < x \circ 0$ for all $x, y \in H$.*

Proof. Since $x \in x \circ 0$ and $x \circ y < x$ for all $x, y \in H$, it is straightforward. $\square$

Proposition 3.3. *In a hyper K-algebra of order 3, the implication* (q1) *is always true.*

Proof. Let $H = \{0, a, b\}$ be a hyper K-algebra of order 3. For the cases $0 < a$ and $0 < b$, it is true by Lemma 3.2. Now assume that $a < b$. Obviously $0 \circ b < 0 \circ a$, $a \circ b < a \circ a$ and $b \circ b < b \circ a$, that is $x \circ b < x \circ a$ for all $x \in H$. Similarly, if $b < a$, then $y \circ b < y \circ a$ for all $y \in H$. This completes the proof. $\square$

Proposition 3.4. *In a bounded hyper K-algebra H with the unit u, we have $x \circ u < x \circ y$ for all $x, y \in H$.*

Proof. Straightforward. $\square$

Proposition 3.5. *Let $H = \{0, a, b, u\}$ be a bounded hyper K-algebra of order 4 with unit u. Then*

(i) $u \circ a = \{a\}$, $a < b \;\Rightarrow\; u \circ b \neq \{b\}$, $u \circ b \neq \{u\}$ *and* $u \circ b \neq \{u, b\}$.
(ii) $b \in u \circ a$, $a < b \Rightarrow u \circ b \neq \{u\}$.

Proof. (i) Assume that $u \circ a = \{a\}$ and $a < b$. Obviously, $u \circ b \neq \{b\}$ and $u \circ b \neq \{u\}$. If $u \circ b = \{b, u\}$, then $0 \in a \circ b = (u \circ a) \circ b = (u \circ b) \circ a = \{b, u\} \circ a = b \circ a \cup u \circ a$, and so $0 \in b \circ a$, i.e., $b < a$, because $0 \notin u \circ a$. This is a contradiction.

(ii) Suppose $u \circ b = \{u\}$. Then $0 \in a \circ b = (u \circ a) \circ b = (u \circ b) \circ a = u \circ a$, and so $u < a$. This is a contradiction, and the proof is complete. $\square$

Proposition 3.6. *In a bounded hyper K-algebra of order 4, the implication* (q1) *is valid.*

Proof. Let $H = \{0, a, b, u\}$ be a bounded hyper K-algebra of order 4 with unit u. Without loss of generality, it is sufficient to show that the result is true for the following three cases:

$$\text{(i) } \forall x \in H, x < u, \text{ (ii) } \forall x \in H, 0 < x \text{ and (iii) } a < b.$$

The first and second cases imply that (q1)is true by Lemma 3.2 and Proposition 3.4. For the last case, we have $h \circ b < h \circ a$ for $h = 0, a, b$, since $0 \in h \circ b$. Now we show that $u \circ b < u \circ a$. If $u \in u \circ a$, then obviously $u \circ b < u \circ a$ because u is the unit. Now, since $0 \notin u \circ a$, we know that $u \circ a = \{a\}$, $u \circ a = \{b\}$, or $u \circ a = \{a, b\}$. Suppose $u \circ a = \{a\}$. Using Proposition 3.5(i), we know that $a \in u \circ b$, and so $u \circ b < u \circ a$ by (H3). Now if $u \circ a = \{b\}$ or $u \circ a = \{a, b\}$, then $u \circ b \neq \{u\}$ by Proposition 3.5(ii). Thus $u \circ b$ contains a or b, and hence $u \circ b < u \circ a$ by hypothesis and (H3). This completes the proof. $\square$

The following example shows that the boundedness in Proposition 3.6 is necessary.

Example 3.7. Let $H = \{0, a, b, c\}$ be a hyper K-algebra in Example 3.1. Then $(H, \circ, 0)$ is not bounded and $a < b$, but $\{c\} = c \circ b \nless c \circ a = \{0, b\}$.

Proposition 3.8. *In a hyper K-algebra H, we have*

$$((x \circ z) \circ (y \circ z)) \circ u < (x \circ y) \circ u, \ \forall x, y, z, u \in H.$$

Proof. For any $x, y, z, u \in H$, we have

$$\begin{aligned} ((x \circ z) \circ (y \circ z)) \circ u &= ((x \circ u) \circ z) \circ (y \circ z) \qquad \text{by (p1)} \\ &= \bigcup_{t \in x \circ u} (t \circ z) \circ (y \circ z) \\ &< \bigcup_{t \in x \circ u} t \circ y \qquad \text{by (H1)} \\ &= (x \circ u) \circ y \\ &= (x \circ y) \circ u. \qquad \text{by (H2)} \end{aligned}$$

This completes the proof. □

In what follows, let H denote a hyper K-algebra unless otherwise specified.

Definition 3.9. [2] A nonempty subset I of H is called a *weak implicative hyper K-ideal* of H if it satisfies (I1) and

(I7) $\forall x, y, z \in H, \ (x \circ z) \circ (y \circ x) \subseteq I, z \in I \Rightarrow x \in I$.

Definition 3.10. [2] A nonempty subset I of H is called an *implicative hyper K-ideal* of H if it satisfies (I1) and

(I8) $\forall x, y, z \in H, \ (x \circ z) \circ (y \circ x) < I, z \in I \Rightarrow x \in I$.

Note that every implicative hyper K-ideal is a weak implicative hyper K-ideal (see [2]).

Definition 3.11. [4] An element a of H is said to be *left* (resp. *right*) *scalar* if $|a \circ x| = 1$ (resp. $|x \circ a| = 1$) for all $x \in H$. If $a \in H$ is both left and right scalar, we say that a is a *scalar element* of H.

The following example shows that there is a hyper K-algebra in which $0 \in H$ may not be a right (resp. left) scalar element.

Example 3.12. Let $H = \{0, a, b\}$ be a set with Cayley table as follows:

$\circ$	0	a	b
0	$\{0, b\}$	$\{0, a, b\}$	$\{0, a, b\}$
a	$\{a, b\}$	$\{0\}$	$\{0, a\}$
b	$\{b\}$	$\{b\}$	$\{0, a, b\}$

Then $(H, \circ, 0)$ is a hyper K-algebra but $0 \in H$ isn't a right (resp. left) scalar element.

Lemma 3.13. *If 0 is a right* (resp. *left*) *scalar element of H, then $x \circ 0 = \{x\}$* (resp. $0 \circ x = \{0\}$) *for all $x \in H$.*

Proof. If follows from (p4) (resp. (H5)) and the fact that 0 is a right (resp. left) scalar element of H. □

Proposition 3.14. *If 0 is a right* (resp. *left*) *scalar element of H, then $A \circ 0 = A$* (resp. $0 \circ A = \{0\}$) *for every nonempty subset A of H.*

Proof. It follows from Lemma 3.13. □

Theorem 3.15. [2] *Every implicative hyper K-ideal is a hyper K-ideal.*

The following examples shows that there is a hyper K-ideal which is not an implicative hyper K-ideal.

Example 3.16. (1) Let $H = \{0, a, b\}$ be a hyper K-algebra in Example 3.12. It is routine to verify that $\{0, a\}$ is a hyper K-ideal, but it is not an implicative hyper K-ideal, since $(b \circ 0) \circ (0 \circ b) = b \circ \{0, a, b\} = \{0, a, b\} < \{0, a\}$ and $0 \in \{0, a\}$ but $b \notin \{0, a\}$.

(2) Let $H = \{0, a, b, c\}$ be a set and consider the following table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, b\}$	$\{a\}$	$\{a, c\}$
b	$\{b\}$	$\{0\}$	$\{0, b\}$	$\{0, b\}$
c	$\{c\}$	$\{0, b\}$	$\{c\}$	$\{0, c\}$

Then $(H, \circ, 0)$ is a hyper K-algebra. Routine calculations, we know that $\{0, b\}$ is a hyper K-ideal, but it is not an implicative hyper K-ideal because $(c \circ 0) \circ (a \circ c) = c \circ \{a, c\} = \{0, b, c\} < \{0, b\}$ and $0 \in \{0, b\}$ but $c \notin \{0, b\}$.

We know that the converse of Theorem 3.15 is not true. It is then natural to ask that is the converse of the Theorem 3.15 true under what condition(s)? Now we solve this question.

Theorem 3.17. *Let I be a non-empty subset of H and 0 a scalar element. Then I is an implicative hyper K-ideal of H if and only if I is a hyper K-ideal and $x \circ (y \circ x) < I$ implies $x \in I$ for all $x, y \in H$.*

Proof. Assume that I is an implicative hyper K-ideal of H. By Theorem 3.15, I is a hyper K-ideal. Let $x, y \in H$ be such that $(x \circ (y \circ x)) < I$. Note that $(x \circ 0) \circ (y \circ x) = x \circ (y \circ x)$. Thus we get $((x \circ 0) \circ (y \circ x)) < I$. Since $0 \in I$, it follows from (I8) that $x \in I$. Conversely, suppose that I is a hyper K-ideal and $x \circ (y \circ x) < I$ implies $x \in I$ for all $x, y \in H$. Let $x, y, z \in H$ be such that $(x \circ z) \circ (y \circ x) < I$ and $z \in I$. Then $(x \circ (y \circ x)) \circ z < I$ and so $u \circ z < I$ for some $u \in x \circ (y \circ x)$. Since I is a hyper K-ideal and $z \in I$, we have $u \in I$ which implies $x \circ (y \circ x) < I$. By hypothesis, we get $x \in I$, which shows that I is a hyper K-ideal. □

Definition 3.18. A non-empty subset I of H is called an s-*implicative hyper K-ideal* of H if it satisfies (I1) and

(I9) $\forall x, y, z \in I,\ ((x \circ z) \circ (y \circ x)) \cap I \neq \emptyset,\ z \in I \ \Rightarrow\ x \in I.$

Example 3.19. Let $H = \{0, a, b\}$ be a set with Cayley table as follows:

$\circ$	0	a	b
0	$\{0\}$	$\{0, a\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, a\}$

Then $(H, \circ, 0)$ is a hyper K-algebra which is not . By routine calculations, we know that $\{0, a\}$ is a (weak) implicative hyper K-ideal and an s-implicative hyper K-ideal. But $I = \{0, b\}$ is a weak implicative hyper K-ideal which is neither an implicative hyper K-ideal nor an s-implicative hyper K-ideal since $(a \circ 0) \circ (0 \circ a) = a \circ \{0, a\} = \{0, a\} < I$ and $((a \circ 0) \circ (0 \circ a)) \cap I \neq \emptyset$, but $a \notin I$.

Theorem 3.20. *Every s-implicative hyper K-ideal is a strong hyper K-ideal.*

Proof. Let I be an s-implicative hyper K-ideal of H and let $x, y \in H$ be such that $y \in I$ and $(x \circ y) \cap I \neq \emptyset$. Then there exists $a \in H$ such that $a \in x \circ y$ and $a \in I$. We have $a \in a \circ 0 \subseteq (x \circ y) \circ (0 \circ x)$. Hence $((x \circ y) \circ (0 \circ x)) \cap I \neq \emptyset$. It follows from (I9) that $x \in I$. Therefore I is a strong hyper K-ideal. □

The converse of Theorem 3.20 may not be true as seen in the following example.

Example 3.21. Let $H = \{0, a, b\}$ be a hyper K-algebra in Example 3.19. Then $\{0, b\}$ is a strong hyper K-ideal. But we know that it is not an s-implicative hyper K-ideal.

In order to consider the converse of Theorem 3.20, we strengthen conditions.

Theorem 3.22. *Let I be a non-empty subset of H and 0 a scalar element. Then I is an s-implicative hyper K-ideal of H if and only if I is a strong hyper K-ideal of H and $(x \circ (y \circ x)) \cap I \neq \emptyset$ implies $x \in I$ for all $x, y \in H$.*

Proof. Assume that I is an s-implicative hyper K-ideal of H. By Theorem 3.20, I is a strong hyper K-ideal. Let $x, y \in H$ be such that $(x \circ (y \circ x)) \cap I \neq \emptyset$. Note that $(x \circ 0) \circ (y \circ x) = x \circ (y \circ x)$. Thus we get $((x \circ 0) \circ (y \circ x)) \cap I \neq \emptyset$. Since $0 \in I$, it follows from (I9) that $x \in I$. Conversely, suppose that I is a strong hyper K-ideal of H and $(x \circ (y \circ x)) \cap I \neq \emptyset$ implies $x \in I$ for all $x, y \in H$. Let $x, y, z \in H$ be such that $((x \circ z) \circ (y \circ x)) \cap I \neq \emptyset$ and $z \in I$. Then $((x \circ (y \circ x)) \circ z) \cap I \neq \emptyset$ and so $(u \circ z) \cap I \neq \emptyset$ for some $u \in x \circ (y \circ x)$. Since I is a strong hyper K-ideal and $z \in I$, we have $u \in I$ and thus $(x \circ (y \circ x)) \cap I \neq \emptyset$. By hypothesis, we get $x \in I$, which shows that I is a strong hyper K-ideal of H. $\square$

Theorem 3.23. *Implicative hyper K-ideals and* s*-implicative hyper K-ideals coincide.*

Proof. Let I be an s-implicative hyper K-ideal of H. Then I is a strong hyper K-ideal of H. Let $x, y, z \in H$ be such that $(x \circ z) \circ (y \circ x) < I$ and $z \in I$. Then there exist $a \in (x \circ z) \circ (y \circ x)$ and $b \in I$ such that $a < b$, i.e., $0 \in a \circ b$. It follows that $(a \circ b) \cap I \neq \emptyset$ so from (I4) that $a \in I$. Hence $((x \circ z) \circ (y \circ x)) \cap I \neq \emptyset$, and so $x \in I$ by (I9). Therefore I is an implicative hyper K-ideal of H. Conversely, Let I be an implicative hyper K-ideal of H and $x, y, z \in H$ such that $((x \circ z) \circ (y \circ x)) \cap I \neq \emptyset$ and $z \in I$. Then there exists $a \in H$ such that $a \in (x \circ z) \circ (y \circ x)$ and $a \in I$. Hence $(x \circ z) \circ (y \circ z) < I$. Since I is an implicative hyper K-ideal, we get $x \in I$. Hence I is an s-implicative hyper K-ideal of H. This completes the proof. $\square$

In general, a weak implicative hyper K-ideal may not be a weak hyper K-ideal as seen in the following example.

Example 3.24. Let $H = \{0, a, b\}$ be a set with Cayley table as follows:

$\circ$	0	a	b
0	$\{0, a, b\}$	$\{0, a, b\}$	$\{0, a, b\}$
a	$\{a\}$	$\{0, a, b\}$	$\{a, b\}$
b	$\{a, b\}$	$\{0, a\}$	$\{0, a, b\}$

Then H is a hyper K-algebra. It is easy to check that $\{0, a\}$ is a weak implicative hyper K-ideal, while it is not a weak hyper K-ideal, because $b \circ a \subseteq \{0, a\}$ and $a \in \{0, a\}$, but $b \notin \{0, a\}$.

We give a condition for a weak implicative hyper K-ideal to be a weak hyper K-ideal.

Theorem 3.25. *If 0 is a scalar element of H, then every weak implicative hyper K-ideal is a weak hyper K-ideal.*

Proof. Let I be a weak implicative hyper K-ideal of H and let $x, y \in H$ be such that $x \circ y \subseteq I$ and $y \in I$. Since 0 is a scalar element, it follows from Proposition 3.14 that $(x \circ y) \circ (0 \circ x) = x \circ y \subseteq I$ so from (I8) that $x \in I$. Hence I is a weak hyper K-ideal of H. $\square$

Proposition 3.26. *For every subsets A and B of H, we have*

$$A < B \Longleftrightarrow 0 \in A \circ B.$$

Proof. It is straightforward by definition of relation "$<$" in hyper K-algebra. $\square$

Proposition 3.27. *A hyper K-algebra H satisfies $x \circ (y \circ x) < x$.*

Proof. Let $x, y \in H$. Then $0 \in 0 \circ (y \circ x) \subseteq (x \circ x) \circ (y \circ x) = (x \circ (y \circ x)) \circ x$ by (I5), (H3) and (H5). It follows from Proposition 3.26 that $x \circ (y \circ x) < x$. $\square$

Definition 3.28. A hyper K-algebra H is said to be *implicative* if

$$x < x \circ (y \circ x) \ \forall x, y \in H.$$

Example 3.29. The hyper K-algebras in Examples 3.12 and 3.19 are implicative, but the hyper K-algebra in Example 3.16(2) is not implicative since $b \not< 0 = b \circ a = b \circ (a \circ b)$.

We give a condition for a hyper K-algebra to be implicative.

Theorem 3.30. *If H is a hyper K-algebra satisfying $x \in x \circ (y \circ x)$ for all $x, y \in H$, then H is implicative.*

Proof. If $x \in x \circ (y \circ x)$ for all $x, y \in H$, then $x < x \circ (y \circ x)$. Hence H is implicative. $\square$

The following example shows that the converse of Theorem 3.30 may not be true.

Example 3.31. Let $H = \{0, a, b\}$ be a set and consider the following table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{0, a\}$	$\{0, a, b\}$

Then $(H, \circ, 0)$ is an implicative hyper K-algebra. But not satisfy $x \in x \circ (y \circ x)$ since $b \notin b \circ (a \circ b) = b \circ a = \{0, a\}$.

Theorem 3.32. *Let I be a reflexive positive implicative hyper K-ideal of H which is of type-1 or type-2. Then $(x \circ y) \circ y \subseteq I$ implies $x \circ y \subseteq I$.*

Proof. If I is of type-1, then it is clear. Assume that I is of type-2. Let $x, y \in H$ such that $(x \circ y) \circ y \subseteq I$. Hence $(x \circ y) \circ y < I$ and $y \circ y \in I$, because I is reflexive. Since I is of type-2, we get $x \circ y \in I$. This completes the proof. $\square$

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