ON ALGEBRAIC CONSTRUCTION OF FAMILY OF ROUGH SETS*

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Abstract. Rough set theory was introduced by Pawlak. It is an excellent tool to handle granularity of data. We know that a field of sets (and fuzzy sets) with the inclusion of sets (fuzzy sets) creates a complete lattice. In this note, we shall discuss when is a family of rough sets with the rough set inclusion a complete lattice. And we shall give its algorithm.

1. Introduction

Rough set theory was introduced by Pawlak [1]. It is an excellent tool to handle granularity of data. During the last 10 years it has attracted the attention of many researchers and practitioners all over the world who contributed to its development and application. Rough set theory may be used to describe dependecies between attributes, to evaluate significance of attributes, and to deal with inconsistent data, to name just a few possible uses of this theory to knowledge and data analysis. As an approuch to handling with uncertain data, such as probability theroy, evidence theory, and fuzzy set set theory,etc ([2]).

The rough set theory is founded on the assumption that, with every object of the universe of discourse, we associate some information. Objects characterized by the same information are indiscernible in view of the available information about them. The indiscernibility relation generated in this way is the mathematical basis for the rough set theory.

It is known that a field of sets with the inclusion of sets creates a complete lattice. For rough sets, when is a family of rough sets with the rough set inclusion a complete lattice?

2. Preliminaries

We assume the following definitions of an approximation space.

Definition 2.1. *The ordered pair $\langle U, \mathcal{C}\rangle$, where U is any nonempty set called a universe, and $\mathcal{C}$ is a finite family of nonempty subsets of U with $\cup\mathcal{C} = U$. We call the ordeded pair an approximation space.*

Definition 2.2 *Let $\langle U, \mathcal{C}\rangle$ be an approximation space, and $x \in U$. Let*

$$Md(x) = \{K \in \mathcal{C} : x \in K \text{ and if } x \in S \in \mathcal{C} \text{ and } S \subset K \text{ then } K = S\}$$

That is, $Md(x)$ is the family of minimal elements containing x in $\mathcal{C}$.

Definition 2.3. *Let $\langle U, \mathcal{C}\rangle$ be an approximation space. For $X \subset U$, we define that $\mathcal{C}_*(X) = \{K \in \mathcal{C} : K \subset X\}$. And $X_* = \cup\mathcal{C}_*(X)$ is called the lower approximation of the set X. $X_*^* = X - X_*$ is the boundary of the set X. Let $Bn(X) = \cup\{Md(x) : x \in X_*^*\}$ and $\mathcal{C}^*(X) = \mathcal{C}_*(X) \cup Bn(X)$. Call $X^* = \cup\mathcal{C}^*(X)$ the upper approximation of X.*

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Definition 2.4. *Let $\langle U, \mathcal{C}\rangle$ be an approximation space. For $x \in U$, define $\mathcal{K}_x = \{K \in \mathcal{C} : x \in K\}$. $\mathcal{K}_x$ is called the neighborhood system of x. It is clear that $\mathcal{K}_x \neq \emptyset$ for each $x \in U$. We call $K_x \in \mathcal{K}_x$ the smallest neighborhood of x, if $K_x \subset K$ for each $K \in \mathcal{K}_x$.*

Remark 2.5. Let $\langle U, \mathcal{C}\rangle$ be an approximation space, and $X \subset U$, then we have the following simple results:

(a). $Bn(X) \cap \mathcal{C}_*(X) = \emptyset$.

(b). If K_1 and K_2 are in $\mathcal{C}$ with $K_1 \neq K_2$, and K_1, K_2 is respectively the smallest neighborhood of x_1 and x_2, then $x_1 \neq x_2$. And if K_1, K_2 is only respectively the smallest neighborhood of x_1 and x_2 with $x_1 \neq x_2$, then $K_1 \neq K_2$.

(c). For any $K \in Bn(X)$, then $K \cap X \neq \emptyset$, and $K \cap (U - X) \neq \emptyset$. That is K contains at least two elements, and K is a minimal element containing some $x \in X^*_*$.

(d). If $K \in Bn(X)$ and K is the smallest neighborhood of x, then $x \notin X_*$.

Definition 2.6. *Let $\langle U, \mathcal{C}\rangle$ be an approximation space. For $X \subset U$, we define $X_{\mathcal{C}} = \{Y : \mathcal{C}_*(Y) = \mathcal{C}_*(X)$ and $\mathcal{C}^*(Y) = \mathcal{C}^*(X)\}$, is called the rough set in $\langle U, \mathcal{C}\rangle$. The family $\{X_{\mathcal{C}} : X \subset U\}$ of all rough sets will be denoted by $Rs(U, \mathcal{C})$.*

Definition 2.7. *For $X_{\mathcal{C}}, Y_{\mathcal{C}} \in Rs(U, \mathcal{C})$, we define that*

$$X_c \subset_c Y_c \Longleftrightarrow \mathcal{C}_*(X) \subset \mathcal{C}_*(Y) \text{ and } \mathcal{C}^*(X) \subset \mathcal{C}^*(Y).$$

The expression $X_c \subset_c Y_c$ should be read: the rough set X_c is roughly included in the rough set Y_c.

It is clear that the relation of rough set inclusion is partial order relation. And we know that a lattice which has only finite elements is a complete lattice.

3. The main results

It is known that a field of sets (fuzzy sets) with the inclusion of sets (fuzzy sets) creates a complete lattice. We have a natural question: When is a family of rough sets with the rough inclusion a complete lattice?

For $X \subset U$, we can get the rough set $X_c \in Rs(U, \mathcal{C})$ is identified with pairs $\langle \mathcal{C}_*(X), \mathcal{C}^*(X)\rangle$. Let $\mathcal{P} = \{\langle \mathcal{C}_*(X), \mathcal{C}^*(X)\rangle : X \subset U\}$. For $\langle A_1, B_1\rangle$, $\langle A_2, B_2\rangle \in \mathcal{P}$, we define that

$$\langle A_1, B_1\rangle \subset_p \langle A_2, B_2\rangle \Longleftrightarrow A_1 \subset A_2 \text{ and } B_1 \subset B_2.$$

We shall find the conditions of existence of supremum and infimum in the poset $\langle \mathcal{P}, \subset_p\rangle$ for any set $\mathcal{A} \subset \mathcal{P}$.

Theorem 3.1. *For an approximation space $\langle U, \mathcal{C}\rangle$, the following conditions are equivalent:*

a). For each $K \in \mathcal{C}$, if $\mid K \mid \geq 2$, then there are at least two elements y and z in U, such that K is the smallest neighborhood of y and z respectively.

b). For any family $\mathcal{A} \subset \mathcal{P}(U)$ (powerset of U), there exists $Z \subset U$, such that

$$\cup\{\mathcal{C}_*(A) : A \in \mathcal{A}\} = \mathcal{C}_*(Z) \qquad \cup\{\mathcal{C}^*(A) : A \in \mathcal{A}\} = \mathcal{C}^*(Z)$$

And there is an $Z_1 \subset U$, such that

$$\cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\} = \mathcal{C}_*(Z_1) \qquad \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\} = \mathcal{C}^*(Z_1)$$

Proof. $a) \Rightarrow b)$. For $\mathcal{A} \subset \mathcal{P}(U)$, let

$$\mathcal{A}_1 = \cup\{\mathcal{C}_*(A) : A \in \mathcal{A}\}, \qquad \mathcal{A}_2 = \cup\{\mathcal{C}^*(A) : A \in \mathcal{A}\}.$$

If $\mathcal{A}_1 = \mathcal{A}_2$, let $Z = \cup\mathcal{A}_1$. It is clear that Z satisfies condition b). If $\mathcal{A}_1 \neq \mathcal{A}_2$, we define Z as follows:

Set $\mathcal{A}_2 - \mathcal{A}_1 = \{K_1, K_2, \cdots, K_p\}$, and $M_i = \{y : K_i$ is the smallest neighborhood of $y\}$ for $1 \leq i \leq p$. We claim the following facts:

(1). If $i \neq j$, then $M_i \cap M_j = \emptyset$. And $| M_i | \geq 2$ because of $| K_i | \geq 2$.

(2). For each $1 \leq i \leq p$, $M_i \cap (\cup \mathcal{A}_1) = \emptyset$. In fact, suppose there is an $y \in U$ with $y \in (M_i \cap (\cup \mathcal{A}_1))$ for some i. Then we can get an $A_y \in \mathcal{A}$ and $K_y \in \mathcal{C}_*(A_y)$ such that $y \in (M_i \cap K_y)$. By the definition of M_i, $K_i \subset K_y$. This contradicts to that $K_i \notin \mathcal{A}_1$.

(3). For each $1 \leq i \leq p$, $K_i \cap (\cup \mathcal{A}) \neq \emptyset$. In fact, since $K_i \in \mathcal{A}_2$, hence $K_i \in \mathcal{C}^*(A)$ for some $A \in \mathcal{A}$, and $K_i \cap A \neq \emptyset$.

(4). Let $Z = \cup \mathcal{A}_1 \cup \{y_1, y_2, \cdots, y_p\}$, where $y_i \in M_i$ for each $1 \leq i \leq p$. It is clear that $\mathcal{A}_1 \subset \mathcal{C}_*(Z)$. We shall prove that $\mathcal{C}_*(Z) \subset \mathcal{A}_1$. Now suppose that $K' \in \mathcal{C}_*(Z)$, i.e., $K' \subset Z$.

(i). If $K' = \{x\}$ for some $x \in U$. By (3), $x \in A_x$ for some $A_x \in \mathcal{A}$, and $\{x\} = K' \subset A_x$. Hence $K' \in \mathcal{C}_*(A_x) \subset \mathcal{A}_1$.

(ii). Suppose $| K' | \geq 2$. From condition a), there are y and z in U with K' is the smallest neighborhood of y and z. If $y \in \cup \mathcal{A}_1$, then we can get some $A_y \in \mathcal{A}$ and $K_y \in \mathcal{C}_*(A_y)$ with $y \in K_y$. Thus $K' \subset K_y$ and $K' \in \mathcal{A}_1$. If $z \in \cup \mathcal{A}_1$, we can get same conclusion by similar method. If $\{y, z\} \cap (\cup \mathcal{A}_1) = \emptyset$, then $\{y, z\} \subset \{y_1, y_2, \cdots, y_p\}$. This contradicts to the construction of $\{y_1, y_2, \cdots, y_p\}$ (see (b) of Remark 2.5).

To sum up we get that $\mathcal{C}_*(Z) \subset \mathcal{A}_1$ and $\mathcal{A}_1 = \mathcal{C}_*(Z)$.

(5). Now we shall prove that $\mathcal{A}_2 = \mathcal{C}^*(Z)$. By means of (1) and (2), $Z^*_* = \{y_1, y_2, \cdots, y_p\}$, and $Bn(Z) = \{K_1, K_2, \cdots, K_p\}$. Hence $\mathcal{A}_2 = \mathcal{C}^*(Z)$.

b) $\Rightarrow$ *a)*. Suppose there were an $K' \in \mathcal{C}$ such that $| K' | \geq 2$ and $| Y | \leq 1$, where $Y = \{y : K' \text{ is the smallest neighborhood of } y\}$.

Suppose $Y = \{a\}$ for some $a \in U$. For each $b \in (K' - Y)$, let $\mathcal{C}_b = \{K : K \in \mathcal{C}$ and $b \in K, K \neq K'\}$. If $\mathcal{C}_b = \emptyset$, then there is only K' in $\mathcal{C}$ which contains b. In this case K' is the smallest neighborhood of b, this is a contradiction. Hence $\mathcal{C}_b \neq \emptyset$ for each $b \in (K' - Y)$. Pick $K_b \in \mathcal{C}_b$ with $K' \not\subset K_b$ (If for any $K_b \in \mathcal{C}_b$, we have that $K' \subset K_b$, then K' is the smallest neighborhood of b, this is a contradiction.) Let $\mathcal{A}' = \{K_b : b \in (K' - Y)$, and $K_b \in \mathcal{C}_b$ with $K' \not\subset K_b\}$. For any $K_b \in \mathcal{A}'$, $a \notin K_b$ because of that $K' \not\subset K_b$ and K' is the smallest neighborhood of a. For the family $\mathcal{A} = \mathcal{A}' \cup \{a\}$, there is a set $Z \subset U$ such that $\mathcal{C}_*(Z) = \cup\{\mathcal{C}_*(A) : A \in \mathcal{A}\}$ by the condition b), hence $\cup \mathcal{A}' \subset Z$ and $K' \notin \mathcal{C}_*(Z)$ (Because of $K' \not\subset A$ for each $A \in \mathcal{A}$.) But by means of condition b),

$\mathcal{C}^*(Z) = \mathcal{C}^*(\{a\}) \cup (\cup\{\mathcal{C}^*(K) : K \in \mathcal{A}'\}) = \{K'\} \cup (\cup\{\mathcal{C}^*(K) : K \in \mathcal{A}'\})$.

Since $(K' - \{a\}) \subset \cup \mathcal{A}' \subset Z$ and $a \notin \cup(\cup\{\mathcal{C}^*(K) : K \in \mathcal{A}'\})$ (if $a \in \cup \mathcal{C}^*(K_b)$ for some $K_b \in \mathcal{A}'$, then $a \in K_b$ and $K' \subset K_b$.), we get that $a \in Z$. Therefore $K' \subset Z$ and $K' \in \mathcal{C}_*(Z)$. This is a contradiction. If $Y = \emptyset$, then we can get also the similar conclusion. Thus a) is true.

Theorem 3.2. *For an approximation space $\langle U, \mathcal{C} \rangle$, the following condition a) implies condition b):*

a). For each $K \in \mathcal{C}$, if $| K | \geq 2$, then there are at least two elements y and z in U, such that K is the smallest neighborhood of y and z respectively.

b). For any family $\mathcal{A} \subset \mathcal{P}(U)$, there is an $Z \subset U$, such that

$$\cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\} = \mathcal{C}_*(Z) \qquad \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\} = \mathcal{C}^*(Z)$$

Proof. *a)* $\Longrightarrow$ *b)*. For any $\mathcal{A} \subset \mathcal{P}(U)$, let

$$\mathcal{B} = \{K_1, K_2, \cdots, K_p\} = \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\} - \cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\},$$

and $M_i = \{y : K_i \text{ is the smallest neighborhood of } y\}$. By (c) of Remark 2.5 and condition a), we know that $| M_i | \geq 2$ for each $1 \leq i \leq p$. Pick out $y_i \in M_i$ and let

$$Z = \cup(\cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\}) \cup \{y_i : 1 \leq i \leq p\}.$$

We prove that $\mathcal{C}_*(Z) = \cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\}$. It is clear that $\cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\} \subset \mathcal{C}_*(Z)$ by means of the definition of Z. Suppose that $K \in \mathcal{C}_*(Z)$, but $K \notin \cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\}$, then we can get an $A_K \in \mathcal{A}$ with $K \notin \mathcal{C}_*(A_K)$, that is $K \not\subset A_K$. Pick out $a \in (K - A_K) \cap \{y_i : 1 \leq i \leq p\}$, there is an $K' \in \mathcal{C}^*(A_K)$ with $a \in K'$, and K' is the smallest neighborhood of a. Since $K' \in Bn(A_K)$ and $K' \subset K$, K' contains at least two elements, hence K has at least two elements. By condition a), there are x and y in U such that K is the smallest neighborhood of x and y and $K \subset Z$.

(1). If either x or y, say that $x \in \cup(\cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\})$. Then for any $A \in \mathcal{A}$, there is an $K_A \in \mathcal{C}_*(A)$ with $x \in K_A$. Since $K \subset K_A$, thus $K \in \cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\}$. This contradicts the mention as above.

(2). If neither x not y is in $\cup(\cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\})$, then $\{x, y\} \subset \{y_1, y_2, \cdots, y_p\}$. This contradicts the struction of $\{y_1, y_2, \cdots, y_p\}$. Therefore $\mathcal{C}_*(Z) = \cap\{\mathcal{C}_*(A) : A \in \mathcal{A}\}$.

(3). Furtherlly we prove that $\mathcal{C}^*(Z) = \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\}$. Suppose that $K \in \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\}$. If $K \in \mathcal{C}_*(A)$ for every $A \in \mathcal{A}$, then $K \in \mathcal{C}_*(Z) \subset \mathcal{C}^*(Z)$ by the above proved part. Else there is an $A'_K \in \mathcal{A}$ with $K \notin \mathcal{C}_*(A'_K)$, then $K \in \mathcal{B}$, i.e., $K = K_i$ for some $1 \leq i \leq p$. Because K_i is the smallest neighborhood of y_i, and $y_i \in Z$, thus $K = K_i \in \mathcal{C}^*(Z)$.

(4). Now we conclude that $\mathcal{C}^*(Z) \subset \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\}$. Suppose there were an $K \in \mathcal{C}^*(Z)$ with $K \notin \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\}$, then $K \in \mathcal{C}^*(Z) - \mathcal{C}_*(Z) = Bn(Z)$. From the definition of Z, we can get an $x \in K \cap \{y_1, y_2, \cdots, y_p\}$. Then $K = K_i$ for some $1 \leq i \leq p$. Hence $K \in \mathcal{B} \subset \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\}$, that is $\mathcal{C}^*(Z) \subset \cap\{\mathcal{C}^*(A) : A \in \mathcal{A}\}$.

Theorem 3.3. *For an approximation space $\langle U, \mathcal{C}\rangle$, the following conditions are equivalent:*

a). For each $K \in \mathcal{C}$, if $\mid K \mid \geq 2$, then there are at least two elements y and z in U, such that K is the smallest neighborhood of y and z respectively.

b). For any family $\mathcal{A} \subset \mathcal{P}(U)$, there is a supremum in the poset $\langle \mathcal{P}, \subset_{\mathcal{P}}\rangle$ as follows
$sup\{\langle \mathcal{C}_*(X), \mathcal{C}^*(X)\rangle : X \in \mathcal{A}\} = \langle \cup\{\mathcal{C}_*(X) : X \in \mathcal{A}\}, \cup\{\mathcal{C}^*(X) : X \in \mathcal{A}\}\rangle$.

c). For any family $\mathcal{A} \subset \mathcal{P}(U)$, there are the following supremum and infimum in the poset $\langle \mathcal{P}, \subset_{\mathcal{P}}\rangle$
$sup\{\langle \mathcal{C}_*(X), \mathcal{C}^*(X)\rangle : X \in \mathcal{A}\} = \langle \cup\{\mathcal{C}_*(X) : X \in \mathcal{A}\}, \cup\{\mathcal{C}^*(X) : X \in \mathcal{A}\}\rangle$,
$inf\{\langle \mathcal{C}_*(X), \mathcal{C}^*(X)\rangle : X \in \mathcal{A}\} = \langle \cap\{\mathcal{C}_*(X) : X \in \mathcal{A}\}, \cap\{\mathcal{C}^*(X) : X \in \mathcal{A}\}\rangle$.

Theorem 3.4. *For an approximation space $\langle U, \mathcal{C}\rangle$, the following conditions are equivalent:*

a). For each $K \in \mathcal{C}$, if $\mid K \mid \geq 2$, then there are at least two elements y and z in U, such that K is the smallest neighborhood of y and z respectively.

b). For any family of rough sets, there is a supremum in a poset $\langle Rs(U, \mathcal{C}), \subset_{\mathcal{C}}\rangle$.

c). For any family of rough sets, there are a supremum and an infimum in a poset $\langle Rs(U, \mathcal{C}), \subset_{\mathcal{C}}\rangle$, that is, the poset $\langle Rs(U, \mathcal{C}), \subset_{\mathcal{C}}\rangle$ is a complete lattice.

4. The algorithm

In this section, we shall give the algorithm of the supremum as above. For the infimum, we can get similarly its algorithm.

The algorithm:

a). Input initial data: sets U, $\mathcal{C}$ and $\mathcal{A}$.
$x = U.getAt(n)$; \\ x is the nth element of U.
$U.insert(a)$; \\ $U \cup \{a\}$.
$U.power()$; \\ retrieves the cardinal number of U.
$U.isIn(A)$; \\ if $A \subset U$, retrieves 1; else retrieves 0.
$A = U.union(B)$; \\ $A = U \cup B$.

$X = \mathcal{A}.selfunin()$; \\ $X = \cup\mathcal{A}$.
$C = A.subtract(B)$; \\ $C = A - B$.
$K = \mathcal{C}.findMin(x)$; \\ K is the smallest element containing x of $\mathcal{C}$.

b). \\ Constructing $\mathcal{A}_1 = \cup\{\mathcal{C}_*(A) : A \in \mathcal{A}\}$.

1). initialize an empty set $\mathcal{A}_1$;
2). $n = \mathcal{A}.power()$;
3). $m = \mathcal{C}.power()$;
4). initialize $i = 1$;
5). initialize $j = 1$;
6). if $\mathcal{C}.getAt(j) \not\subset \mathcal{A}.getAt(i)$, goto 8);
7). $\mathcal{A}_1.insert(\mathcal{C}.getAt(j))$;
8). $j = j + 1$; if $j \leq m$, goto 6);
9). $i = i + 1$; if $i \leq n$, goto 5);
10). $W = \mathcal{A}_1.selfunion()$;

c). \\ Constructing $\mathcal{A}_2$.

1). initialize an empty set X_*;
2). initialize an empty set $\mathcal{A}_2$;
3). $n = \mathcal{A}.power()$;
4). $m = \mathcal{C}.power()$;
5). initialize $i = 1$;
6). initialize $j = 1$;
7). if $\mathcal{C}.getAt(j) \not\subset \mathcal{A}.getAt(i)$, goto 10);
8). $X_* = X_*.union(\mathcal{C}.getAt(j))$;
9). $\mathcal{A}_2.insert(\mathcal{C}.getAt(j))$;
10). $j = j + 1$; if $j \leq m$, goto 7);
11). $X_*^* = \mathcal{A}.getAt(i).subtract(X_*)$;
12). $p = X_*^*.power()$;
13). initialize $k = 1$;
14). $\mathcal{A}_2.insert(\mathcal{C}.findMin(X_*^*.getAt(k)))$;
15). $k = k + 1$; if $k \leq p$, goto 14);
16). $i = i + 1$; if $i \leq n$, goto 6);

d). \\ Constructing $Y = \{y_1, y_2, \cdots, y_p\}$.

1). $V = \mathcal{A}_2.subtract(\mathcal{A}_1)$;
2). initialize an empty set Y;
3). $n = V.power()$;
4). initialize $i = 1$;
5). $m = V.getAt(i).power()$;
6). initialize $j = 1$;
7). if $\mathcal{C}.findMin(V.getAt(i).getAt(j)) = V.getAt(i)$; then $Y.insert(V.getAt(i).$ $getAt(j))$; and goto 9);
8). $j = j + 1$; if $j \leq m$, goto 7);
9). $i = i + 1$; if $i \leq n$, goto 5);

e). \\ Constructing Z.

1). initialize an empty set Z;
2). $Z = W.union(Y)$;

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