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THE RANKING OF TOTAL MEDALS WON BY EACH COUNTRY IN THE SYDNEY OLYMPICS BY USING DEA MODEL AFTER CLUSTERING

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Abstract. When one tries to sequence a large number of candidates from the voting data by using DEA model, he/she often observes the fact that it is extremely difficult for low-vote gainers to take full advantage of DEA model. In addition, one encounters a problem of sequencing a huge number of candidates on a scale of '1'. One can overcome such problems, however, by sequencing candidates in each cluster with DEA model after breaking down the candidates into clusters. In this paper, the authors make two proposals as measures as such. One is to stratify the candidates in advance so as to make clear the sequence among the clusters. The other is to seek the alignment between clusters rank order relations and within cluster rank order relations of the candidates.

1 Introduction When the Most Valuable Player (MVP) is selected in a Japanese professional baseball league, each of the voters (designated sports journalists) holds a three-time voting right to order the ranking, the first to the third, from the list of eligible players as candidates. 5 points are allotted to the first ranking, 3 to the second, and 1 to the third respectively, and the MVP is selected according to the result of total MAD (multiply and add). However, if 4 points are allotted to the second ranking and a certain player is ranked as such by many voters, then, it is often the case that this particular player comes from behind and wins an advantage over the MVP who has been previously chosen. Such a problem in point-allotment is called " the problem of weighting". Cook and Kress proposed a measure to automatically decide on the rank order weight in order for each candidate to hold the advantage using Data Envelopment Analysis (DEA) model (Cook & Kress (1990) [1]). Later, Green and Cook et al. evolved the measure so as to make it possible to decide on the total rank order of all candidates (Green, Doyle & Cook (1996) [2]). Let us call this particular measure, 'Green's Measure' in this paper. However, Noguchi and Ishii found out the following problems in the behavior of each rank. That is, there are cases in which the weights become the same in the different ranks, and in which the weight of a certain rank becomes 0, and it seems voteless, or as if it has not been voted at all. Accordingly, Noguchi and Ishii introduced a new weighting measure to overcome such problems (Noguchi & Ishii (2000) [3]). Furthermore, they have presented a ranking measure that enables the extension of its multipurpose applications rather than a single-purpose application (Noguchi & Ishii (2000) [4]). From these researches, they have concluded that the ranking measure with DEA model, in the case that the total number of candidates is small, has been successfully established. Nonetheless, it is true, when there are a great number of candidates, that the comparisons are made among those whose differences in the votes are quiet large, and that the weights of ranks are fixed on the basis of the candidate whose vote gain is the greatest.

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Therefore, the candidates whose vote gains are relatively low cannot take full advantage of DEA model since it is difficult to set up the weight in order to keep their edges. Also, what happens, when the ordering is done at the same time among a large number of candidates, is that they have to be laid out on the very short scale of '1'; therefore, the sequencing process itself becomes complicated.

In this paper, the measures to overcome the above problems are speculated in the example of the number of medals (gold, silver, bronze) each country won in the Sydney Olympic Games. The total number of countries that won medals counted eighty; here such countries are called candidates for our evaluation. Problems above can be solved when candidates are broken down into clusters on a certain basis prior to the sequencing of each cluster by the use of DEA. There are two ways to categorizing or breaking the candidates into clusters. One way of doing it is to sequence candidates in each cluster after stratifying so as to make the order among clusters clearly distinct; let us call this procedure 'Classification I'. The other is to break down into several clusters by the use of cluster analysis beforehand and then seek the alignment between cluster rank order relations and within cluster rank order relations of the candidates; let us call this procedure 'Classification II'. In Chapter 2, we will discuss methodologies with the total ranking of medals at the Sydney Olympic Games. In Chapter 3, we will present the result of the medal ranking and the validity of our measures. Finally in Chapter 4, we will present our summaries of this paper.

2 The Measures of Categorizing Each Country and Its Sequence When one tries to sequence all the eighty nations that won medals by the use of DEA model, what happens is the fact that those nations, whose total number of medals is small, are forced to incur the constraints of the nations whose number of medals is the largest. In such a case, for those nations whose medals are few, DEA model will not be advantageous because it is difficult to set up the weights that give them advantage. Therefore, it will be much easier to set up the advantageous weights if the sequencing is done in every cluster after breaking down the nations into clusters in accordance with the situations. Classifications I and II are presented in the following sections as two procedures for the purpose of breaking down the nations in accordance with the number of medals; then the measure of sequence by the use of DEA is presented at the end of this Chapter.

2.1 Classification I : The stratifying of Each Country Classification I is the measure to stratify after making the order distinctions clear among the clusters. This is to do the ordering until the order of medal gains of each country becomes clear, and then to stratify at the point where the ordering becomes no longer possible. As shown in Chart 1, let us represent the referent nations by m ($m = 1, 2, \ldots, M$), then the total medal gains by y_{mk} in each rank k ($k = 1, 2, \ldots, K : K = 3$ since we deal with three types of medals —gold(G), silver(S), and bronze(B)—at this time). In this case, the order of total medal gains can be fixed as in the following procedure.

(a) Sort the countries in accordance with the number of medal gains

(b) Compare Country m with Country q ($m, q = 1, 2, \ldots, M, m \neq q$) by each medal rank. If $y_{\mathsf{m1}} \geq y_{\mathsf{q1}}$, $y_{\mathsf{m2}} \geq y_{\mathsf{q2}}, \ldots, y_{\mathsf{mk}} \geq y_{\mathsf{qk}}$ (α), then $\therefore m >= q$ ($\therefore m >= q$, which means Country m is ranked higher than Country q or they are ranked the same)

(c) Or if any of the greater-than signs is reversed, then pick up the sign at the lowest medal rank at that point. Then, it is ranked as t ($t = 1 \sim K$). $y_{\mathsf{mt}} < y_{\mathsf{qt}}$; therefore, $y_{\mathsf{qt}} - y_{\mathsf{mt}} = T_{\mathsf{m<q,t}}$. Next, pick up $(t-1)$ of the rank just above t:

$$y_{\mathsf{qt-1}} + T_{\mathsf{m<q,t}} = y'_{\mathsf{qt-1}}. \text{ Then, compare } y'_{\mathsf{qt-1}} \text{ with } y_{\mathsf{mt-1}}. \text{ If}$$
$$y_{\mathsf{m1}} \geq y_{\mathsf{q1}}, \quad y_{\mathsf{m2}} \geq y_{\mathsf{q2}}, \ldots, y_{\mathsf{mt-l}} \geq y'_{\mathsf{qt-1}} \quad (\beta), \text{ then } \therefore m >= q\ .$$

Chart 1: Medal Ranks and Total Medal Gains (y_{mk})

Countries	Medal Ranks					
	1	2	$\cdots$	k	$\cdots$	K
1	y_{11}	y_{12}	$\cdots$	$y_{1\mathsf{k}}$	$\cdots$	$y_{1\mathsf{K}}$
2	y_{21}	y_{22}	$\cdots$	$y_{2\mathsf{k}}$	$\cdots$	$y_{2\mathsf{K}}$
$\cdots$			$\cdots$		$\cdots$	
m	$y_{\mathsf{m}1}$	$y_{\mathsf{m}2}$	$\cdots$	y_{mk}	$\cdots$	y_{mK}
$\cdots$			$\cdots$		$\cdots$	
M	$y_{\mathsf{M}1}$	$y_{\mathsf{M}2}$	$\cdots$	y_{Mk}	$\cdots$	y_{MK}

(d) Again, if reversed greater-than signs are still found when Country m is compared with Country q, pick up the sign at the lowest medal rank. Then, it is ranked as s ($s \leq t-1$). $y_{\mathsf{ms}} < y_{\mathsf{qs}}$; therefore, $y_{\mathsf{qs}} - y_{\mathsf{ms}} = T_{\mathsf{m<q,s}}$. Next, pick up $(s-1)$ of the rank just above s : $y_{\mathsf{qs-1}} + T_{\mathsf{m<q,s}} = y'_{\mathsf{qs-1}}$. Then as in (c), compare $y'_{\mathsf{qs-1}}$ with $y_{\mathsf{ms-1}}$. If $y_{\mathsf{m1}} \geq y_{\mathsf{q1}}$, $y_{\mathsf{m2}} \geq y_{\mathsf{q2}}, \cdots, y_{\mathsf{ms-1}} \geq y'_{\mathsf{qs-1}}$ (γ), then, $\therefore m >= q$.
(e) In the case that the greater-than signs are still found reversed, then, pick up the one at the lowest medal rank. Then it is ranked as h. Repeat (c) and (d) until the reversed greater signs are absorbed into the one just above : $(h \leq s-1)$. If $y_{\mathsf{m1}} \geq y'_{\mathsf{q1}}$ finally, then, $\therefore m >= q$.
(f) Do the sequencing procedure by repeating (a)$\sim$(e) from the two countries whose medal gains are the greatest.
(g) If the sequencing is no longer possible at $y_{\mathsf{m1}} < y'_{\mathsf{q1}}$ eventually, then, a lump emerges. Extract such a lump and group the countries, in which the sequencing has not been impossible, as one cluster.
(h) Finally, do the sequencing using DEA model at every cluster.

Classification I, as shown above, is the method in which the sequencing is done from the two countries whose total number of medal gains are the greatest, after stratifying the countries into clusters.

2.2 Classification II : Sorting by the Use of Cluster Analysis Classification II is to directly employ a sorting technique. Cluster Analysis is know as one of the typical techniques. There are many types of Cluster Analyses: the variety of the set-ups of classification criteria and the distances of objects, hierarchical ones in which the sequential objects are merged one by one, and non-hierarchical ones in which the number of clusters is fixed prior to its classification. Here in this paper, K-means, a non-hierarchical technique, is employed because it is to classify, among a great number of countries, at different cutoff points that vary along with the situations of medal gains. Compared with the methods in which grouping is done hierarchically among the similar individuals, K-means is to optimize the dividing so as to minimize the sum of squares of each individual's distance within clusters while maximizing those between the clusters. Therefore, K-means is appropriate in such a case as this in which the sorting optimization is intended while setting up the initial number of medals of each cluster in accordance with the medal gains among the gold, the silver, and the bronze. Specific operational expressions of it are omitted here since it is not the main purpose of this paper. A great number of operations are repeatedly done. Then, the calculation is stopped at the point where the sum of squares of mean distances between the clusters are found to remain stably maximum while those within clusters stay minimum even if individual entries are interchanged. K-means is known as a method that leads to stable results when a great number of entries are classified (MacQueen, J. (1967) [6]). The sequence of clusters is determined by the average medal gains in the clusters. Next, in order to align the sequence of each country within-cluster with those of between the top and the next clusters, pick up the countries commonly found in both clusters. That is, the countries that are ranked low in the upper cluster are aligned with the same countries ranked high in the lower cluster. Then, in the case that the sequence alignment of some countries cannot be made, they are to be grouped as another cluster, in which sequencing will be done by the use of DEA (The result $= \Omega$). The sequencing procedure is completed when Ω is inserted between the upper ranked countries in the upper cluster and those ranked low the lower cluster.

2.3 The Sequencing by using DEA model after Classifying Here, Green's Measure is summarized as a sequencing procedure by the use of DEA. The total number of countries who won medals is to be M. Then, the total number of medals (by medal ranks) is y_{mk}. w_{mk} (the most preferable weight for a certain country m) is set so as to maximize the MAD of y_{mk} and w_{mk}. Then the productivity θ_{mm} of Country m is shown as in (1).

$$\theta_{mm} = \text{Maximize} \sum_{k=1}^{K} w_{mk} y_{mk} \tag{1}$$

Here, $m = 1, 2, \ldots, M$. Also, $k = 1, 2, \ldots, K$. Then, $y_{m1}, y_{m2}, \ldots,$ or y_{mk} represents the medal gains at a certain medal rank (K) won by Country m. Stable sequencing will not take place without any constraints on w_{mk}, because w_{mk} can be set unlimitedly large for the purpose of increasing θ_{mm}. Therefore, Green, et al. set (2) and (3) as constraints on w_{mk} when enlarging θ_{mm} of each country .

$$\theta_{mq} = \sum_{k=1}^{K} w_{mk} y_{qk} \leq 1 \quad (q = 1, 2, \ldots, M) \tag{2}$$

on the condition of

$$w_{mk} - w_{mk-1} \geq d(k-1, \varepsilon) = \varepsilon \geq 0, \quad w_{m1} \geq w_{m2} \geq \ldots \geq w_{mk} \geq 0 \tag{3}$$

$d(k-1, \varepsilon) = \varepsilon$ in (3) represents the difference of the rank weights between $k-1$ and k. It is possible for each to set its own w_{mk} on preferable terms when (2) and (3) are combined. θ_{mq} is to be obtained for each country. Then, each θ_{mq} is distributed into all the elements ($M \times M$ rows) ; the sequence of medal gains among M countries will be determined by arranging decreasing order of the added average in each row, according to Green's Measure.

Noguchi et al., however, pointed out its inapplicable cases in which different ranks ($i-1$ and i) are not distinguishable (the values of gold and silver medals are seen as the same) because of $w_{mi-1} = w_{mi}$ or when the weight of rank i is ignored (the bronze values are not counted) because of $w_{mi} = 0$, if (3) remains as it is (Noguchi & Ishii (2000) [3]). So Noguchi and Ishii set (4) in place of (3). The constraint is the laxest one with which the weighting is possible at each medal rank, and θ_{mq} can be obtained without fail when a certain country wins any kind of medals. Whereas, θ_{mq} with DEA is represented in ratio. So Noguchi et al employed the Geometrical Mean (GM) when final rankings of M countries are calculated from the θ_{mq} of $M \times M$ rows because θ_{mq} is a ratio scaling.

$$w_{mk} - w_{mk-1} \geq d(k-1, \varepsilon) = \varepsilon \geq 0.0001, \quad w_{m1} > w_{m2} > \ldots > w_{mk} > 0 \tag{4}$$

In this paper, a single criterion of selection, θ_{mq} or the total medal gains, is analyzed; however, other criteria of selection, such as fairness of athletes in each country, can also be analyzed. In such a case Noguchi's Measure (Noguchi & Ishii (2000) [5]) is applicable.

3 The Ranking of Medal Gains by Each Country at Sydney Olympics

Chart 2: Medal Gains by Countries (on the basis of gold medal totals)
G : the number of gold medals, S : that of silver medals, B : that of bonze medals

No Country	G S B	No Country	G S B	No Country	G S B
1 America	39 25 33	28 Czech Republic	2 3 3	55 Belgium	0 2 3
2 Russia	32 28 28	29 Kenya	2 3 2	56 South Africa	0 2 3
3 China	28 16 15	30 Denmark	2 3 1	57 Argentina	0 2 2
4 Australia	16 25 17	31 Finland	2 1 1	58 Morocco	0 1 4
5 Germany	14 17 26	32 Austria	2 1 0	59 Taiwan	0 1 4
6 France	13 14 11	33 Lithuania	2 0 3	60 North Korea	0 1 3
7 Italy	13 8 13	34 Azerbaijan	2 0 1	61 Saudi Arabia	0 1 1
8 Netherlands	12 9 4	35 Slovenia	2 0 0	62 Moldova	0 1 1
9 Cuba	11 11 7	36 Switzerland	1 6 2	63 Trinidad and	
10 Britain	11 10 7	37 Indonesia	1 3 2	Tobago	0 1 1
11 Romania	11 6 9	38 Slovakia	1 3 1	64 Ireland	0 1 0
12 South Korea	8 9 11	39 Mexico	1 2 3	65 Uruguay	0 1 0
13 Hungary	8 6 3	40 Algeria	1 1 3	66 Vietnam	0 1 0
14 Poland	6 5 3	41 Uzbekistan	1 1 2	67 Georgia	0 0 6
15 Japan	5 8 5	42 Latvia	1 1 1	68 Costa Rica	0 0 2
16 Bulgaria	5 6 2	43 Yugoslavia	1 1 1	69 Portugal	0 0 2
17 Greece	4 6 3	44 Bahamas	1 1 0	70 Armenia	0 0 1
18 Sweden	4 5 3	45 New Zealand	1 0 3	71 Barbados	0 0 1
19 Norway	4 3 3	46 Estonia	1 0 2	72 Chile	0 0 1
20 Ethiopia	4 1 3	47 Thailand	1 0 2	73 India	0 0 1
21 Ukraine	3 10 10	48 Croatia	1 0 1	74 Iceland	0 0 1
22 Kazakhstan	3 4 0	49 Cameroon	1 0 0	75 Israel	0 0 1
23 Belarus	3 3 11	50 Colombia	1 0 0	76 Kirgyzstan	0 0 1
24 Canada	3 3 8	51 Mozambique	1 0 0	77 Kuwait	0 0 1
25 Spain	3 3 5	52 Brazil	0 6 6	78 Macedonia	0 0 1
26 Iran	3 0 1	53 Jamaica	0 4 3	79 Qatar	0 0 1
27 Turkey	3 0 1	54 Nigeria	0 3 0	80 Srilanka	0 0 1

Hitler took full advantage of the Olympics, a global-scale festivity, in order to enhance nationalism or to raise consciousness among its own nationals toward the emerging Nazism. He also devised the torch relay from Greece; the medal gains in the Olympics have been considered important, ever since, for participating countries to appeal their dignities to the world. As known as they are, the medals are divided into three ranks: gold(G), silver(S), and bronze(B). The sequence or ranking of medal gains among the participating countries has been beyond one's power since it can vary considerably depending upon the weights set on those three ranks. Therefore, Noguchi and Ishii, this time, try to present the measure for sequencing the countries by the use of DEA model that enables each country to set weights on the three ranks that are preferable for itself. However, when sequencing is done at one time among as many as eighty countries with DEA model, the weights of the ranks for all the participating countries are set on the basis of America whose total medal gains are the greatest. In such a case like this, then, there are three patterns for weighting that emerge: (a) $w_{\mathrm{m1}} = 0.0104$ (gold), $w_{\mathrm{m2}} = 0.0103$ (silver), and $w_{\mathrm{m3}} = 0.0102$ (bronze); (b) $w_{\mathrm{m1}} = 0.0254$, $w_{\mathrm{m2}} = 0.0002$, $w_{\mathrm{m3}} = 0.0001$; (c) $w_{\mathrm{m1}} = 0.0156$, $w_{\mathrm{m2}} = 0.0155$, $w_{\mathrm{m3}} = 0.0001$ respectively. In any of these patterns, θ_{mm} of America becomes 1.0000. The weighting becomes (a) for countries whose medal gains among the three ranks are approximately the same and those whose bronze medal gains are great. There are 63 countries that fall into this category. Next, the weighting becomes (b) for those whose gold medal gains stand out. There are eight countries as such. Finally the weighting becomes (c) for those whose gold and silver medal gains are high while their bronze gains are relatively low. These countries amount to five. In either case above, the weights are small, and they are set by the country whose total medal gain is the largest among all the participants. Therefore, the preferable weighting, for those whose medal gains are few, will be severely restricted. Also, it is problematic to sequence them, since θ_{mm} of these countries too become extremely small.

However, such problems as above can be avoided if the participating countries are divided into clusters prior to sequencing countries whose medal gains are on the same levels. The following section shows the results as such; then Noguchi and Ishii will make out the world ranking of the medal gains at the Sydney Olympics, fin de siecle.

3.1 The Rank Order of Medal Gains after Executing Classification I As shown in 2.1, analyze countries into clusters. Then, the sequencing among the top 6 countries is sequentially done as a result. The sequence of these six countries as Group A is shown as in Chart 3.

Chart 3 : The Sequence of Countries in Group A by Classification I

No Country	G S B	Rank Order
1 America	39 25 33	1
2 Russia	32 28 28	2
3 China	28 16 15	3
4 Australia	16 25 17	4
5 Germany	14 17 26	5
6 France	13 14 11	6

(Now, G :Gold , S : Silver, B : Bronze)

Chart 4 : The Sequence of Countries in Group B by Classification I
(GM is Geometrical Mean and θ_{mm} is the productivity of m's country)

No Country	G S B	Each weight of each medal (G/S/B) 0.0295 0.0294 0.0293	0.0432 0.0431 0.0072	0.0544 0.0365 0.0001	0.0767 0.0002 0.0001	GM of θ_{mm} And Rank Order
7 Italy	13 8 13	1.0000	1.0000	1.0000	1.0000	1.0000 1
9 Cuba	11 11 7	0.8533	1.0000	1.0000	0.8466	0.9225 2
10 Britain	11 10 7	0.8239	0.9569	0.9635	0.8464	0.8940 3
12 South Korea	8 9 11	0.8232	0.8127	0.7643	0.6165	0.7637 6
11 Romania	11 6 9	0.7649	0.7987	0.8179	0.8458	0.8011 5
8 Netherlands	12 9 4	0.7361	0.9355	0.9812	0.9226	0.8755 4
Deriving equation of GM		$X^{2/6}$	$Y^{1/6}$	$Z^{2/6}$	$U^{1/6}$	$= A$

Next, when Country 6 (France: 13 golds; 14 silvers; 11 bronzes) is compared with Country 7 (Italy: 13 golds; 8 silvers; 13 bronzes), $y_{F1}(=13) \geq y_{I1}(=13)$, $y_{F2}(=14) \geq y_{I2}(=8)$, $y_{F3}(=11) \leq y_{I3}(=13)$ can be obtained by (c) in 2.1. Then, $y_{I3}(=13) - y_{F3}(=11) = T_{F<I,3}(=2)$, $y_{I2}(=8) + T_{F<I,3}(=2) = y'_{I2}(=10)$, $y_{F2}(=14) \geq y'_{I2}(=10)$. Then, France is ranked above Italy by (β) of (c) in 2.1. If the sequencing is continued in the same way, the rank orders from Country 7 (Italy) to Country 12 (South Korea) in Chart 2 cannot be decided. However, all these six countries can be ranked above Country 13 (Hungary) by the use of sequencing in 2.1, so they are to be grouped as one cluster or as Group B. The sequence of these six countries by the use of DEA model can be shown as in Chart 4. Countries from 13 and below cannot be divided into groups; therefore, these remaining 68 countries should be categorized together as Group C. Chart 5 shows the result of sequencing within Groups C by the use of DEA model. Here, there are five weighting patterns, so that there are wider selections for preferable weights for each country. In addition, the weights are about four times as large as those of when eighty countries are compared at one time, so the sequencing becomes easier. In Classification I, the sequence of Groups A,

B, C is A>B>C. Therefore, the rank order of medal gains at Sydney will be completed by sequencing countries within each group according to Chart 3 to 5. The Ranking Class-I in Chart 6 shows the final result by using this measure.

Chart 5 : The Sequence of the Countries in Group C by Classification I
(GM is Geometrical Mean and θ_{mm} is the productivity of m's country)

No Country	G S B	Each weight of each medal (G/S/B)					GM of θ_{mm} And Rank Order
		0.0436	0.0693	0.0827	0.0715	0.1248	
		0.0435	0.0692	0.0376	0.0714	0.0002	
		0.0434	0.0100	0.0375	0.0001	0.0001	
21 Ukraine	3 10 10	1.0000	1.0000	1.0000	0.9289	0.3774	0.9308 1
15 Japan	5 8 5	0.7832	0.9504	0.9023	0.9286	0.6262	0.8668 3
13 Hungary	8 6 3	0.7402	1.0000	1.0000	1.0000	1.0000	0.9366 2
23 Belarus	3 3 11	0.7389	0.5253	0.7740	0.4295	0.3761	0.6303 7
14 Poland	6 5 3	0.6094	0.7921	0.7970	0.7856	0.8492	0.7528 4
24 Canada	3 3 8	0.6086	0.4954	0.6614	0.4292	0.3758	0.5609 9
16 Bulgaria	5 6 2	0.5659	0.7820	0.7144	0.7856	0.6255	0.6937 5
17 Greece	4 6 3	0.5659	0.7227	0.6693	0.7142	0.5007	0.6484 6
18 Sweden	4 5 3	0.5222	0.6534	0.6316	0.6429	0.5005	0.6025 8
52 Brazil	0 6 6	0.5215	0.4752	0.4512	0.4287	0.0018	0.3266 17
25 Spain	3 3 5	0.4784	0.4655	0.5487	0.4289	0.3758	0.4881 11
19 Norway	4 3 3	0.4352	0.5139	0.5563	0.5002	0.5001	0.5089 10
36 Switzerland	1 6 2	0.3915	0.5047	0.3837	0.4798	0.1261	0.3897 14
20 Ethiopia	4 1 3	0.3482	0.3765	0.4810	0.3575	0.4997	0.4126 12
28 Czech	2 3 3	0.3480	0.3763	0.3910	0.3571	0.2505	0.3636 15
22 Kazakhstan	3 4 0	0.3049	0.4850	0.3986	0.4996	0.3752	0.4006 13
29 Kenya	2 3 2	0.3046	0.3663	0.3534	0.3572	0.2504	0.3376 16
53 Jamaica	0 4 3	0.3043	0.3068	0.2632	0.2857	0.0011	0.1982 25
30 Denmark	2 3 1	0.2612	0.3564	0.3159	0.3571	0.2504	0.3106 18
37 Indonesia	1 3 2	0.2610	0.2970	0.2707	0.2857	0.1256	0.2622 19
39 Mexico	1 2 3	0.2609	0.2377	0.2706	0.2145	0.1255	0.2420 20
67 Georgia	0 0 6	0.2605	0.0598	0.2253	0.0006	0.0006	0.0621 42
33 Lithuania	2 0 3	0.2174	0.1686	0.2780	0.1432	0.2499	0.2184 23
38 Slovakia	1 3 1	0.2175	0.2870	0.2332	0.2856	0.1255	0.2359 22
40 Algeria	1 1 3	0.2173	0.1685	0.2330	0.1431	0.1253	0.1948 26
55 Belgium, 56 S. Africa	0 2 3	0.2172	0.1683	0.1879	0.1430	0.0007	0.1276 33
58 Moroc, 59 Taiwan	0 1 4	0.2171	0.1091	0.1878	0.0718	0.0006	0.1067 37
26 Iran, 27 Turkey	3 0 1	0.1742	0.2180	0.2856	0.2145	0.3745	0.2386 21
31 Finland	2 1 1	0.1741	0.2179	0.2406	0.2144	0.1251	0.2077 24
41 Uzbekistan	1 1 2	0.1739	0.1585	0.1954	0.1430	0.1251	0.1709 28
45 New Zealand	1 0 3	0.1738	0.0982	0.1953	0.0718	0.1251	0.1429 31
57 Argentina	0 2 2	0.1738	0.1584	0.1504	0.1429	0.0006	0.1089 36
60 N. Korea	0 1 3	0.1737	0.0991	0.1503	0.0717	0.0005	0.0901 40
32 Austria	2 1 0	0.1307	0.2079	0.2030	0.2143	0.2498	0.1891 27
34 Azerbaijan	2 0 1	0.1306	0.1486	0.2029	0.1430	0.2497	0.1682 29
42 Latvia, 43 Yugo	1 1 1	0.1305	0.1485	0.1579	0.1429	0.1251	0.1456 30
46 Estonia, 47 Thai	1 0 2	0.1304	0.0893	0.1578	0.0717	0.1250	0.1209 34
54 Nigeria	0 3 0	0.1305	0.2077	0.1129	0.2141	0.0006	0.1014 39
35 Slovenia	2 0 0	0.0872	0.1387	0.1654	0.1429	0.2496	0.1400 32
44 Bahamas	1 1 0	0.0871	0.1386	0.1203	0.1428	0.1250	0.1181 35
48 Croatia	1 0 1	0.0870	0.0793	0.1202	0.1202	0.1249	0.1026 38
61 SaudiAr, 62 Moldova, 63 Trinidad Tobago	0 1 1	0.0869	0.0792	0.0752	0.0715	0.0003	0.0544 43
68 CostaRi, 69 Portug	0 0 2	0.0868	0.0199	0.0750	0.0002	0.0002	0.0483 44
49 Cam, 50 Col, 51 Moz	1 0 0	0.0436	0.0693	0.0827	0.0715	0.1248	0.0700 41
64 Ireland, 65 Uruguay, 66 Vietnam	0 1 0	0.0435	0.0692	0.0376	0.0714	0.0002	0.0338 45
70 Arme ... 80 Srila	0 0 1	0.0434	0.0099	0.0375	0.0001	0.0001	0.0103 46
Deriving equation of GM		$X^{10/46}$	$Y^{10/46}$	$Z^{18/46}$	$U^{5/46}$	$V^{3/46}$	$= A$

Chart 6 : The ranking of medal gains by each country at the Sydney Olympics obtained by Classification I and Classification II (following section 3.2)

No Country	G	S	B	The Ranking Class-I	Class-II	No Country	G	S	B	The Ranking Class-I	Class-II
1 America	39	25	33	1	1	41 Uzbekistan	1	1	2	41	42
2 Russia	32	28	28	2	2	34 Azerbaijan	2	0	1	42	39
3 China	28	16	15	3	3	42 Latvia	1	1	1	43	43
4 Australia	16	25	17	4	4	43 Yugoslavia	1	1	1	43	43
5 Germany	14	17	26	5	5	45 New Zealand	1	0	3	45	46
6 France	13	14	11	6	6	35 Slovenia	2	0	0	46	41
7 Italy	13	8	13	7	7	55 Belgium	0	2	3	47	52
9 Cuba	11	11	7	8	8	56 South Africa	0	2	3	47	52
10 Britain	11	10	7	9	9	46 Estonia	1	0	2	49	48
8 Netherlands	12	9	4	10	10	47 Thailand	1	0	2	49	48
11 Romania	11	6	9	11	11	44 Bahamas	1	1	0	51	47
12 South Korea	8	9	11	12	12	57 Argentina	0	2	2	52	57
21 Ukraine	3	10	10	13	14	58 Morocco	0	1	4	53	58
13 Hungary	8	6	3	14	13	59 Taiwan	0	1	4	53	58
15 Japan	5	8	5	15	15	54 Nigeria	0	3	0	55	51
14 Poland	6	5	3	16	16	60 North Korea	0	1	3	56	60
16 Bulgaria	5	6	2	17	17	48 Croatia	1	0	1	57	50
17 Greece	4	6	3	18	18	49 Cameroon	1	0	0	58	54
23 Belarus	3	3	11	19	19	50 Colombia	1	0	0	58	54
18 Sweden	4	5	3	20	20	51 Mozambique	1	0	0	58	54
24 Canada	3	3	8	21	21	67 Georgia	0	0	6	61	64
19 Norway	4	3	3	22	22	61 Saudi Arabia	0	1	1	62	61
25 Spain	3	3	5	23	23	62 Moldova	0	1	1	62	61
20 Ethiopia	4	1	3	24	24	63 Trinidad and					
22 Kazakhstan	3	4	0	25	25	Tobago	0	1	1	62	61
36 Switzerland	1	6	2	26	27	68 Costa Rica	0	0	2	65	68
28 Czech Republic	2	3	3	27	28	69 Portugal	0	0	2	65	68
29 Kenya	2	3	2	28	29	64 Ireland	0	1	0	67	65
52 Brazil	0	6	6	29	26	65 Uruguay	0	1	0	67	65
30 Denmark	2	3	1	30	30	66 Vietnam	0	1	0	67	65
37 Indonesia	1	3	2	31	33	70 Armenia	0	0	1	70	70
39 Mexico	1	2	3	32	37	71 Barbados	0	0	1	70	70
26 Iran	3	0	1	33	31	72 Chile	0	0	1	70	70
27 Turkey	3	0	1	33	31	73 India	0	0	1	70	70
38 Slovakia	1	3	1	35	35	74 Iceland	0	0	1	70	70
33 Lithuania	2	0	3	36	36	75 Israel	0	0	1	70	70
31 Finland	2	1	1	37	34	76 Kirgyzstan	0	0	1	70	70
53 Jamaica	0	4	3	38	45	77 Kuwait	0	0	1	70	70
40 Algeria	1	1	3	39	40	78 Macedonia	0	0	1	70	70
32 Austria	2	1	0	40	38	79 Qatar	0	0	1	70	70
						80 Srilanka	0	0	1	70	70

3.2 The Rank Order of Medal Gains after Executing Classification II The rank order between Group A and B, obtained by Classification I, is clear, so let us keep it as it is. We will further analyze Group C now, using Classification II. Here, Group C—with 46 medal gain patterns, or in total 68 countries—is analyzed with k-means. The parentheses indicate the number of divided groups, and Group C is being divided into five groups in up to the fifth classification attempt. In Chart 7, Attempt 5(5) for example, represents five divided groups at the fifth attempt, and the chart shows the transition of the total sum of squares of both between and within clusters in the five attempts. When sorted by the size of this total sum squares, its most appropriate turnout point, or bifurcation, for classification, would be the one where the oblique angles turn from sharp to lax. According to this chart, then, Attempt 2(2) is to be the most appropriate for the division. As a result, Group C is now divided into Group C1 (13 countries) and Group C2 (45 countries).

Chart 7 : Cluster Analysis: Transition of the total sum squares by the use of k-means

Attempt 1(1) = 851, Attempt 2(2) = 419, . . . , Attempt 5(5) = 278

If the average value of medal gains in C1 is compared with that of C2, gold = 3.77, silver = 5.38, and bronze = 4.92 in C1, and gold = 0.80, silver = 0.93, and bronze = 1.51 in C2 are obtained. Therefore, C1 is ranked above C2 in the between cluster comparison. However, the rank order relation becomes unclear because there are some countries in C2 that could be ranked above Brazil (52) in C1. So, prior to sequencing within C1, we add Ethiopia (20), Kazakhstan (22), Czech (28), Iran (26), and Turkey (27) in C2 to C1. Chart 8 shows the result of the sequencing after the addition of these countries.

Chart 8 : Classification II : The rank order of countries by Cluster Analysis

No Country	G S B	Each weight of each medal (G/S/B)					GM of θ_{mm} And Rank Order
		0.0827	0.0693	0.0715	0.0436	0.1248	
		0.0376	0.0692	0.0714	0.0435	0.0002	
		0.0375	0.0100	0.0001	0.0434	0.0001	
C1 13 Hungary	8 6 3	1.0000	1.0000	1.0000	0.7402	1.0000	0.9652 1
C1 14 Poland	6 5 3	0.7970	0.7921	0.7858	0.6094	0.8492	0.7727 4
C1 15 Japan	5 8 5	0.9023	0.9504	0.9286	0.7832	0.6262	0.8822 3
C1 16 Bulgaria	5 6 2	0.7144	0.7822	0.7856	0.5659	0.6255	0.7125 5
C1 17 Greece	4 6 3	0.6693	0.7227	0.7142	0.5657	0.5007	0.6587 6
C1 18 Sweden	4 5 3	0.6316	0.6534	0.6429	0.5222	0.5005	0.6154 8
C1 19 Norway	4 3 3	0.5563	0.5139	0.5002	0.4352	0.5001	0.5206 10
C1 21 Ukraine	3 10 10	1.0000	1.0000	0.9289	1.0000	0.3774	0.9361 2
C1 23 Belarus	3 3 11	0.7740	0.5253	0.4295	0.7389	0.3761	0.6284 7
C1 24 Canada	3 3 8	0.6614	0.4954	0.4292	0.6086	0.3758	0.5625 9
C1 25 Spain	3 3 5	0.5487	0.4655	0.4289	0.4784	0.3755	0.4935 11
C1 36 Switzerland	1 6 2	0.3837	0.5047	0.4998	0.3915	0.1261	0.3963 14
C1 52 Brazil	0 6 6	0.4512	0.4952	0.4287	0.5215	0.0018	0.3337 16
28 Czech	2 3 3	0.3910	0.3763	0.3571	0.3480	0.2505	0.3684 15
26 Iran, 27 Turkey	3 0 1	0.2856	0.2180	0.2145	0.1742	0.3745	0.2484 17
22 Kazakhstan	3 4 0	0.3986	0.4850	0.4998	0.3049	0.3752	0.4139 13
20 Ethiopia	4 1 3	0.4810	0.3765	0.3575	0.3472	0.4997	0.4229 12
Deriving equation of GM		$X^{8/17}$	$Y^{4/17}$	$Z^{2/17}$	$U^{2/17}$	$V^{1/17}$	$= A$

Five weighting patterns for medal ranks emerge, and the choice of weights on medal ranks remains the same as in Classification I. Therefore, the sequence in the lower ranking is: Ethiopia(20) $>$ Kazakhstan(22) $>$ Switzerland(36) $>$ Czech(28) $>$ Brazil(52) $>$

Iran(26)/Turkey(27). It should be noted that Ethiopia and Kazakhstan are ranked above Switzerland and Czech although their medal gains are fewer. Also, considering the total medal gains among gold, silver, and bronze, Ethiopia and Kazakhstan should be ranked just under Canada(24). And so, Ethiopia and Kazakhstan are included in Group C1; the rank order down to Switzerland is determined. Next, the rest—Czech, Brazil, Iran, and Turkey—are added to Group C2 for further sequencing. Chart 9 shows the result of it. The options for weighting amount to seven patterns now, the choice for preferable weighting is now freer than before. The higher ranking in Group C2 is: Czech(28) > Kenya(29) > Denmark(30) > Iran(26)/Turkey(27) > Indonesia(37) > Finland(31) > Brazil(52). Here, we try to align the rank order between the low-ranked countries in C1 and the high-ranked countries in C2 with Brazil as an interposer, the country common in both these two groups. Brazil is ranked under Czech and above Iran and Turkey in Group C1; whereas, it is ranked seventh under Iran and Turkey in Chart 9. As a result, Czech, Brazil, Iran, Turkey, Kenya, Denmark, Indonesia, and Finland should be seen as not yet aligned, judging from Chart 9. Also, the rank order below Slovakia(38) can be easily determined, which means that the rank order for all the countries can be determined if these eight unaligned countries are sequenced with DEA. The result is shown in Chart 10.

Chart 9: Classification II: The ranking of the countries in C2 with Brazil

No Country	G S B	Each weight of each medal (G/S/B)							GM of θ_{mm} And Rank Order
		0.3214 0.0833 0.0357	0.2500 0.1167 0.0500	0.2500 0.1167 0.0001	0.3330 0.1103 0.0001	0.3333 0.0002 0.0001	0.0835 0.0834 0.0833	0.1667 0.1666 0.0001	
26 Iran, 27 Tur	3 0 1	1.0000	0.8000	0.7531	0.9991	0.9991	0.3337	0.5001	0.6502 4
28 Czech	2 3 3	1.0000	1.0000	0.8503	0.9972	0.6675	0.6670	0.8332	0.8379 1
29 Kenya	2 3 2	0.9643	0.9500	0.9999	0.9971	0.6674	0.5837	0.8331	0.8131 2
30 Denmark	2 3 1	0.9286	0.9000	0.9998	0.9970	0.6673	0.5004	0.8330	0.7683 3
31 Finland	2 1 1	0.7619	0.6667	0.7665	0.7777	0.6669	0.3336	0.4999	0.5695 6
32 Austria	2 1 0	0.7262	0.6167	0.7664	0.7776	0.6668	0.2503	0.4998	0.5168 11
33 Lithuania	2 0 3	0.7500	0.6500	0.5003	0.6663	0.6668	0.4168	0.3336	0.5406 9
34 Azerbaijan	2 0 1	0.6786	0.5500	0.5001	0.6661	0.6667	0.2502	0.3334	0.4510 12
35 Slovenia	2 0 0	0.6429	0.5000	0.5000	0.6660	0.6666	0.1670	0.3333	0.3942 14
37 Indonesia	1 3 2	0.6429	0.7000	0.7499	0.6641	0.3334	0.5002	0.6666	0.5953 5
38 Slovakia	1 3 1	0.6071	0.6500	0.7498	0.6640	0.3340	0.4169	0.6665	0.5547 8
39 Mexico	1 2 3	0.5952	0.6333	0.4837	0.5548	0.3340	0.5001	0.5001	0.5225 10
40 Algeria	1 1 3	0.5119	0.5167	0.3670	0.4445	0.3338	0.4167	0.3335	0.4262 13
41 Uzbekistan	1 1 2	0.4762	0.4667	0.3669	0.4444	0.3337	0.3334	0.3334	0.3903 15
42 Latvia, 43	1 1 1	0.4405	0.4167	0.3668	0.4443	0.3336	0.2501	0.3333	0.3561 16
44 Bahamas	1 1 0	0.4048	0.3667	0.3667	0.4433	0.3335	0.1669	0.3333	0.3029 19
45 NewZeal	1 0 3	0.4286	0.4000	0.2503	0.3333	0.3336	0.3333	0.1670	0.3296 18
46 Estonia, 47	1 0 2	0.3929	0.3500	0.2502	0.3332	0.3335	0.2500	0.1669	0.2894 20
48 Croatia	1 0 1	0.3571	0.3000	0.2501	0.3331	0.3334	0.1668	0.1668	0.2176 21
49 Camer $\sim$ 51	1 0 0	0.3214	0.2560	0.2500	0.3330	0.3333	0.0835	0.1667	0.1971 24
52 Brazil	0 6 6	0.7143	1.0000	0.7006	0.6624	0.0018	1.0000	1.0000	0.5683 7
53 Jamaica	0 4 3	0.4405	0.6167	0.4670	0.4415	0.0011	0.5834	0.6666	0.3544 17
54 Nigeria	0 3 0	0.2500	0.3500	0.3500	0.3309	0.0006	0.2501	0.4997	0.2040 22
55 Belg, 55	0 2 3	0.2738	0.3833	0.2336	0.2209	0.0007	0.4166	0.3334	0.1981 23
57 Algentina	0 2 2	0.2381	0.3333	0.2335	0.2208	0.0006	0.3333	0.3333	0.1895 25
58 Moroc, 59	0 1 4	0.2262	0.3167	0.1170	0.1107	0.0006	0.4165	0.1670	0.1568 26
60 N. Korea	0 1 3	0.1905	0.2667	0.1169	0.1106	0.0005	0.3332	0.1669	0.1377 27
61 Saud $\sim$ 63	0 1 1	0.1190	0.1667	0.1168	0.1104	0.0003	0.1667	0.1667	0.0947 28
64 Irel $\sim$ 66	0 1 0	0.0833	0.1167	0.1167	0.1103	0.0002	0.0834	0.1666	0.0247 30
67 Georgia	0 0 6	0.2143	0.3000	0.0006	0.0006	0.0006	0.4997	0.0006	0.0320 29
68 CostaRi, 69	0 0 2	0.0714	0.1000	0.0002	0.0002	0.0002	0.1666	0.0002	0.0086 31
70 Arme $\sim$ 80	0 0 1	0.0357	0.0500	0.0001	0.0001	0.0001	0.0833	0.0001	0.0043 32
Deriving equation of GM		$X^{7/32}$	$Y^{3/32}$	$Z^{4/32}$	$U^{4/32}$	$V^{2/32}$	$T^{9/32}$	$S^{3/32}$	$= A$

This way, the ranking of medal gains by all the countries can be determined by placing the rank order obtained in Chart 10 after Switzerland(36), ranked fourteenth in Chart 8, and after that, placing the rank order of countries obtained in Chart 9, from Slovakia. This final result is shown at Class-II in the above Chart 6.

Chart 10 : Classification: The ranking of the lower-ranked countries in C1 and the higher-ranked countries in C2

No Country	G S B	Each weight of each medal (G/S/B)			GM of θ_{mm} And Rank Order
		0.0835 0.0834 0.0833	0.2500 0.0834 0.0833	0.2500 0.1666 0.0001	
52 Brazil	0 6 6	1.0000	1.0000	1.0000	1.0000 1
28 Czech	2 3 3	0.6670	1.0000	1.0000	0.9438 2
26 Iran, 27 Turkey	3 0 1	0.3337	0.8333	0.7501	0.6990 5
29 Kenya	2 3 2	0.5837	0.9167	0.9999	0.8921 3
30 Denmark	2 3 1	0.5004	0.8334	0.9998	0.8377 4
31 Finland	2 1 1	0.3336	0.6667	0.6667	0.6039 7
37 Indonesia	1 3 2	0.5002	0.6667	0.7499	0.6730 6
Deriving equation of GM		$X^{1/7}$	$Y^{3/7}$	$Z^{3/7}$	$= A$

The rank order correlation between the ranking result by Classification I and that by Classification II in Chart 6 is 0.994 by Spearman's rank correlation method, which means that its level of significance is 1%; therefore, it should be adopted that these two ranking result remain approximately the same. It is advisable, then, to employ Classification I when it comes to close sequencing, and Classification II in the case that having the edge is necessary for countries whose medal gains are few.

4 Conclusion We have observed that DEA model can be fully employed when one wants to set weights on different ranks in order for each country to preferably have the edge, as in the Olympic games. However, when it comes to sequencing among many, one is forced to compare the countries whose total medal gains extremely differ; therefore, the weights on ranks are determined on the basis of the country whose medal gains are the greatest. Furthermore, sequencing itself will be difficult because it is done at one time while a great number of countries (or candidates) are measured on the scale of '1'. These drawbacks can be overcome if sequencing is done with DEA after classifying countries into groups based by a certain standard. This way, preferable weighting on medal ranks becomes easier for each country, and there will be as many scales as groups; as a result, sequencing countries with few medal gains becomes easier as well. There are two classification measures of this kind. One (Classification I) is to sequence on the premise that the weights of medal ranks are gold > silver >bronze, and that the rank order can be determined in accordance with the number of total medal gains. Later, further sequencing is done among the countries as a group(s), whose rank orders have not been determined in the previous procedure, by the use of DEA. The other (Classification II) is to classify countries into groups in advance by the use of Cluster Analysis. Then, the between-cluster rank orders are determined by the average medals gains of each cluster (group), and sequencing is processed from the higher ranked clusters. Finally, the alignment of rank order relations, both between and within clusters, is sought after by placing the countries commonly listed in both upper and lower clusters. Classification I is a sequencing procedure that is acquired a uniform and reliable rank order. Whereas, Classification II is the one that can lay out well aligned sequence with some rank order variations in accordance with the various classifications. We have implemented two sequencing procedures in the medal gains of participating countries at the Sydney Olympics.

We have found, in either case, that sequencing is better equipped with DEA, which means that DEA model functions beneficially for countries with few medal gains. In sequencing by the use of DEA model, we believe, these two procedures, Classification I and II, should be applied to practical use for making best use of DEA model.

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