APPROXIMATION OF COMMON FIXED POINTS OF A FAMILY OF FINITE NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we deal with an iterative scheme for finding common fixed points of a family of finite nonexpansive mappings in a Banach space. We extend a result of Bauschke in a Hilbert space to a Banach space and a result of Shioji and Takahashi for a single nonexpansive mapping to a family of finite mappings.

1. Introduction

Let C be a closed convex subset of a Banach space E. A mapping $T : C \to C$ is called nonexpansive if $\|Tx - Ty\| \le \|x - y\|$ for all $x, y \in C$. We denote by $F(T)$ the set of fixed points of T. We deal with the iterative process: $x_0 \in C$ and

$$x_{n+1} = \alpha_{n+1}x_0 + (1 - \alpha_{n+1})T_{n+1}x_n, \ n = 0, 1, 2, \dots ,$$

where $T_1, T_2, \dots , T_r$ are nonexpansive mappings of C into itself, $T_{n+r} = T_n$ and $0 < \alpha_{n+1} < 1$. In 1992, Wittmann [11] dealt with the iterative process for $r = 1$ in a Hilbert space and obtained a strong convergence theorem for finding a fixed point of the mapping; see originally Halpern [3]. Shioji and Takahashi [7] extended the result of Wittmann to a Banach space. On the other hand, in 1996, Bauschke [1] dealt with the iterative process for finding a common fixed point of finite nonexpansive mappings in a Hilbert space; see also Lions [5].

The objective of this paper is to obtain a strong convergence theorem which unifies the results by Bauschke [1] and Shioji and Takahashi [7]. Then, using this result, we consider the problem of image recovery in a Banach space setting.

2. Preliminaries

Throughout this paper, all vector spaces are real. Let E be a Banach space and let E^* be its dual. The value of $f \in E^*$ at $x \in E$ will be denote by $\langle x, f\rangle$. We denote by I the identity mapping on E and by J the duality mapping of E into 2^{E^*}, i.e., $Jx = \{f \in E^* \mid \langle x, f\rangle = \|x\|^2 = \|f\|^2\}$, $x \in E$. Let $U = \{x \in E \mid \|x\| = 1\}$. A Banach space E is said to be strictly convex if $\|x + y\|/2 < 1$ for $x, y \in U$. In a strictly convex Banach space, we have that if $\|x\| = \|y\| = \|(1 - \lambda)x + \lambda y\|$ for $x, y \in E$ and $0 < \lambda < 1$, then $x = y$. For every ϵ with $0 \le \epsilon \le 2$, we define the modulus $\delta(\epsilon)$ of convexity of E by

$$\delta(\epsilon) = \inf\left\{1 - \frac{\|x + y\|}{2} \mid \|x\| \le 1, \|y\| \le 1, \|x - y\| \ge \epsilon\right\}.$$

A Banach space E is said to be uniformly convex if $\delta(\epsilon) > 0$ for every $\epsilon > 0$. A Banach space E is said to be smooth provided

$$\lim_{t\to 0} \frac{\|x + ty\| - \|x\|}{t}$$

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exists for each $x, y \in U$. The norm of E is said to be uniformly Gâteaux differentiable if, for each $y \in U$, the above limit exists uniformly for $x \in U$. It is known that if E is smooth then the duality mapping J is single-valued. Moreover it is known that if the norm of E is uniformly Gâteaux differentiable then the duality mapping is norm to weakstar, uniformly continuous on each bounded subset of E.

Let C be a closed convex subset of E and let F be a subset of C. A mapping P of C onto F is said to be sunny if $P(Px + t(x - Px)) = Px$ for each $x \in C$ and $t \geq 0$ with $Px + t(x - Px) \in C$. A subset F of C is said to be a nonexpansive retract of C if there exists a nonexpansive retraction of C onto F. We know the following [6] (see also [9]):

Theorem 2.1. *Let E be a uniformly convex Banach space whose norm is uniformly Gâteaux differentiable, let C be a closed convex subset of E and let T be a nonexpansive mapping of C into itself with $F(T) \neq \emptyset$. Let $x_0 \in C$ and let z_t be a unique element of C which satisfies $z_t = tx_0 + (1-t)Tz_t$ and $0 < t < 1$. Then $\{z_t\}$ converges strongly to a fixed point of T as $t \to 0$. Moreover $\langle x_0 - y, J(y - z)\rangle \geq 0$ for all $z \in F(T)$. Further if $Px_0 = \lim_{t\to 0} z_t$ for each $x_0 \in C$, then P is a sunny nonexpansive retraction of C onto $F(T)$.*

We use this result in the proof of Theorem 3.2.

Let μ be a continuous linear functional on l^∞ and let $(a_0, a_1, \dots) \in l^\infty$. We write $\mu_n(a_n)$ instead of $\mu((a_0, a_1, \dots))$. We call μ a Banach limit when μ satisfies $\|\mu\| = \mu_n(1) = 1$ and $\mu_n(a_{n+1}) = \mu_n(a_n)$ for all $(a_0, a_1, \dots) \in l^\infty$. Let μ be a Banach limit. Then $\liminf_{n\to\infty} a_n \leq \mu(a) \leq \limsup_{n\to\infty} a_n$ for each $(a_0, a_1, \dots) \in l^\infty$. Specially, if $a_n \to p$, then $\mu(a) = p$; see [8] for more details.

3. Strong Convergence Theorem

In this section, we give our main theorem. Before giving it, we prove the following:

Proposition 3.1. *Let E be a strictly convex Banach space and let C be a closed convex subset of E. Let $S_1, S_2, \dots, S_r$ be nonexpansive mappings of C into itself such that the set of common fixed points of $S_1, S_2, \dots, S_r$ is nonempty. Let $T_1, T_2, \dots, T_r$ be mappings of C into itself given by $T_i = (1-\lambda_i)I + \lambda_i S_i$, $0 < \lambda_i < 1$ for each $i = 1, 2, \dots, r$. Then $\{T_1, T_2, \dots, T_r\}$ satisfies $\bigcap_{i=1}^r F(T_i) = \bigcap_{i=1}^r F(S_i)$ and*

$$\bigcap_{i=1}^{r} F(T_i) = F(T_r T_{r-1} \cdots T_1) = F(T_1 T_r \cdots T_2) = \cdots = F(T_{r-1} \cdots T_1 T_r).$$

Proof. For simplicity, we give the proof of Proposition for $r = 2$. It is clear that $F(S_1) \cap F(S_2) = F(T_1) \cap F(T_2)$ and $F(T_1) \cap F(T_2) \subset F(T_2 T_1)$. We prove that $F(S_1) \cap F(S_2) \supset F(T_2 T_1)$. Let z be in $F(T_2 T_1)$ and let w be a common fixed point of S_1 and S_2. Since

$$\begin{aligned}
\|z - w\| &= \|\lambda_2 S_2[(1-\lambda_1)z + \lambda_1 S_1 z] + (1-\lambda_2)[(1-\lambda_1)z + \lambda_1 S_1 z] - w\| \\
&\leq \lambda_2 \|S_2[(1-\lambda_1)z + \lambda_1 S_1 z] - w\| + (1-\lambda_2)\|(1-\lambda_1)z + \lambda_1 S_1 z - w\| \\
&\leq \|(1-\lambda_1)z + \lambda_1 S_1 z - w\| \\
&\leq (1-\lambda_1)\|z - w\| + \lambda_1 \|S_1 z - w\| \\
&\leq \|z - w\|,
\end{aligned}$$

we have

$$\begin{aligned}
\|z - w\| &= \|\lambda_2 S_2[(1-\lambda_1)z + \lambda_1 S_1 z] + (1-\lambda_2)[(1-\lambda_1)z + \lambda_1 S_1 z] - w\| \\
&= \|S_2[(1-\lambda_1)z + \lambda_1 S_1 z] - w\| \\
&= \|(1-\lambda_1)z + \lambda_1 S_1 z - w\| \\
&= \|S_1 z - w\|.
\end{aligned}$$

By the strict convexity of E, we have $S_2[(1-\lambda_1)z+\lambda_1 S_1 z]-w=(1-\lambda_1)z+\lambda_1 S_1 z-w$ and $z-w=S_1 z-w$. Therefore we obtain that $z=S_1 z=S_2 z$. This completes the proof. $\square$

Now we state our main result.

Theorem 3.2. *Let E be a uniformly convex Banach space whose norm is uniformly Gâteaux differentiable and let C be a closed convex subset of E. Let $T_1, T_2, \dots, T_r$ be nonexpansive mappings of C into itself such that the set $F=\bigcap_{i=1}^{r} F(T_i)$ of common fixed points of $T_1, T_2, \dots, T_r$ is nonempty and*

$$\bigcap_{i=1}^{r} F(T_i)=F(T_r T_{r-1}\cdots T_1)=F(T_1 T_r\cdots T_2)=\cdots=F(T_{r-1}\cdots T_1 T_r).$$

Let $\{\alpha_n\}\subset(0,1)$ be a sequence which satisfies $\lim_{n\to\infty}\alpha_n=0$, $\sum_{n=1}^{\infty}\alpha_n=\infty$ and $\sum_{n=1}^{\infty}|\alpha_{n+r}-\alpha_n|<\infty$. Define a sequence $\{x_n\}$ by $x_0\in C$ and

$$x_{n+1}=\alpha_{n+1}x_0+(1-\alpha_{n+1})T_{n+1}x_n,\ n=0,1,2,\dots,$$

where $T_{n+r}=T_n$. Then $\{x_n\}$ converges strongly to a point z in F. Further, if $Px_0=\lim_{n\to\infty}x_n$ for each $x_0\in C$, then P is a sunny nonexpansive retraction of C onto F.

Proof. We first show that $\lim_{n\to\infty}\|x_{n+r}-x_n\|=0$. Since $F\neq\emptyset$, $\{x_n\}$ and $\{T_{n+1}x_n\}$ are bounded. Then there exists $L>0$ such that $\|x_{n+r}-x_n\|\le L|\alpha_{n+r}-\alpha_n|+(1-\alpha_{n+r})\|x_{n+r-1}-x_{n-1}\|$ for each $n=1,2,\dots$. Therefore we have

$$\begin{aligned}\|x_{n+r}-x_n\| &\le L\sum_{k=m+1}^{n}|\alpha_{k+r}-\alpha_k|+\|x_{m+r}-x_m\|\prod_{k=m+1}^{n}(1-\alpha_{k+r})\\ &\le L\sum_{k=m+1}^{n}|\alpha_{k+r}-\alpha_k|+\|x_{m+r}-x_m\|\exp\Big(-\sum_{k=m+1}^{n}\alpha_{k+r}\Big)\end{aligned}$$

for all $n\ge m$. This yields $\limsup_{n\to\infty}\|x_{n+r}-x_n\|\le L\sum_{k=m+1}^{\infty}|\alpha_{k+r}-\alpha_k|$ by $\sum_{k=1}^{\infty}\alpha_k=\infty$. Hence by $\sum_{k=1}^{\infty}|\alpha_{k+r}-\alpha_k|<\infty$, we obtain $\lim_{n\to\infty}\|x_{n+r}-x_n\|=0$. Next we prove $\lim_{n\to\infty}\|x_n-T_{n+r}\cdots T_{n+1}x_n\|=0$. It suffices to show that $\lim_{n\to\infty}\|x_{n+r}-T_{n+r}\cdots T_{n+1}x_n\|=0$. Since $x_{n+r}-T_{n+r}x_{n+r}=\alpha_{n+r}(x_0-T_{n+r}x_{n+r-1})$ and $\lim_{n\to\infty}\alpha_n=0$, we have $x_{n+r}-T_{n+r}x_{n+r-1}\to 0$. From

$$\begin{aligned}\|x_{n+r}-T_{n+r}T_{n+r-1}x_{n+r-2}\| &\le \|x_{n+r}-T_{n+r}x_{n+r-1}\|\\ &\quad+\|T_{n+r}x_{n+r-1}-T_{n+r}T_{n+r-1}x_{n+r-2}\|\\ &\le \|x_{n+r}-T_{n+r}x_{n+r-1}\|\\ &\quad+\|x_{n+r-1}-T_{n+r-1}x_{n+r-2}\|\\ &= \|x_{n+r}-T_{n+r}x_{n+r-1}\|\\ &\quad+\alpha_{n+r-1}\|x_0-T_{n+r-1}x_{n+r-2}\|,\end{aligned}$$

we also have $x_{n+r}-T_{n+r}T_{n+r-1}x_{n+r-2}\to 0$. Similarly, we obtain the conclusion.

Let z_t^n be a unique element of C which satisfies $0<t<1$ and $z_t^n=tx_0+(1-t)T_{n+r}T_{n+r-1}\cdots T_{n+1}z_t^n$. From $F(T_{n+r}T_{n+r-1}\cdots T_{n+1})=F$ and Theorem 2.1, $\{z_t^n\}$ converges strongly to Px_0 of as $t\downarrow 0$, where P is a sunny nonexpansive retraction of C onto F. We show $\limsup_{n\to\infty}\langle x_0-Px_0, J(x_n-Px_0)\rangle\le 0$. Let $A=\limsup_{n\to\infty}\langle x_0-Px_0, J(x_n-Px_0)\rangle$. Then there exists a subsequence $\{x_i\}$ of $\{x_n\}$ such that $A=\lim_{i\to\infty}\langle x_0-Px_0, J(x_{n_i}-Px_0)\rangle$. We assume that $n_i\equiv k \pmod r$ for some $k\in\{1,2,\dots,r\}$. Since

$$\begin{aligned}\|x_{n_i}-T_{n_i+r}\cdots T_{n_i+1}z_t^k\|^2 &\le [\|x_{n_i}-T_{n_i+r}\cdots T_{n_i+1}x_{n_i}\|\\ &\quad+\|T_{n_i+r}\cdots T_{n_i+1}x_{n_i}-T_{n_i+r}\cdots T_{n_i+1}z_t^k\|]^2\\ &\le [\|x_{n_i}-T_{n_i+r}\cdots T_{n_i+1}x_{n_i}\|+\|x_{n_i}-z_t^k\|]^2\end{aligned}$$

and $\|x_{n_i} - T_{n_i+r}\cdots T_{n_i+1}x_{n_i}\| \to 0$, we have

$$\mu_i\|x_{n_i} - T_{n_i+r}\cdots T_{n_i+1}z_t^k\|^2 \le \mu_i\|x_{n_i} - z_t^k\|^2.$$

From $(1-t)(x_{n_i} - T_{n_i+r}\cdots T_{n_i+1}z_t^k) = (x_{n_i} - z_t^k) - t(x_{n_i} - x_0)$, we have

$$\begin{aligned}(1-t)^2\|x_{n_i} - T_{n_i+r}\cdots T_{n_i+1}z_t^k\|^2 &\ge \|x_{n_i} - z_t^k\|^2 - 2t\langle x_{n_i} - x_0, J(x_{n_i} - z_t^k)\rangle \\ &= (1-2t)\|x_{n_i} - z_t^k\|^2 \\ &\quad +2t\langle x_0 - z_t^k, J(x_{n_i} - z_t^k)\rangle\end{aligned}$$

for each i. These inequalities yield

$$\mu_i\langle x_0 - z_t^k, J(x_{n_i} - z_t^k)\rangle \le \frac{t}{2}\mu_i\|x_{n_i} - z_t^k\|^2.$$

As t tends to 0, we obtain

$$\mu_i\langle x_0 - Px_0, J(x_{n_i} - Px_0)\rangle \le 0$$

because E has a uniformly Gâteaux differentiable norm. Hence we have

$$\begin{aligned}A &= \lim_{i\to\infty}\langle x_0 - Px_0, J(x_{n_i} - Px_0)\rangle \\ &= \mu_i\langle x_0 - Px_0, J(x_{n_i} - Px_0)\rangle \le 0.\end{aligned}$$

Now we can prove $\{x_n\}$ converges strongly to Px_0. Let $\epsilon > 0$. By $\limsup_{n\to\infty}\langle x_0 - Px_0, J(x_n - Px_0)\rangle \le 0$, there exists a positive integer n_0 such that

$$\langle x_0 - Px_0, J(x_n - Px_0)\rangle \le \frac{\epsilon}{2}$$

for all $n \ge n_0$. Since $(1-\alpha_n)(T_nx_{n-1} - Px_0) = (x_n - Px_0) - \alpha_n(x_0 - Px_0)$, we have

$$\begin{aligned}(1-\alpha_n)^2\|T_nx_{n-1} - Px_0\|^2 &\ge \|x_n - Px_0\|^2 - 2\alpha_n\langle x_0 - Px_0, J(x_n - Px_0)\rangle \\ &\ge \|x_n - Px_0\|^2 - \alpha_n\epsilon\end{aligned}$$

for all $n \ge n_0$. This yields

$$\|x_n - Px_0\|^2 \le (1-\alpha_n)\|T_nx_{n-1} - Px_0\|^2 + \alpha_n\epsilon$$

for all $n \ge n_0$. Then we have

$$\|x_n - Px_0\|^2 \le \Big(\prod_{k=n_0+1}^{n}(1-\alpha_k)\Big)\|x_{n_0} - Px_0\|^2 + \Big\{1 - \prod_{k=n_0+1}^{n}(1-\alpha_k)\Big\}\epsilon.$$

Hence by $\sum_{k=1}^{\infty}\alpha_k = \infty$, we obtain $\limsup_{n\to\infty}\|x_n - Px_0\|^2 \le \epsilon$. Since ϵ is arbitrary positive real number, $\{x_n\}$ converges strongly to Px_0. $\square$

Remark. When E is a Hilbert space, Theorem 3.2 is the result of Bauschke.

4. Applications to the Feasibility Problem

In this section, we deal with strong convergence theorems which are connected with the feasibility problem. Using a nonlinear ergodic theorem, Crombez [2] considered the feasibility problem in a Hilbert space setting. Let H be a Hilbert space, let $C_1, C_2, \ldots, C_r$ be closed convex subsets of H and let I be the identity operator on H. Then the feasibility problem in a Hilbert space setting may be stated as follows: The original (unknown) image z is known a priori to belong to the intersection C_0 of r well-defined sets $C_1, C_2, \ldots, C_r$ in a Hilbert space; given only the metric projections P_{C_i} of H onto C_i $(i = 1, 2, \ldots, r)$, recover z by an iterative scheme. Crombez [2] proved the following: Let $T = \alpha_0 I + \sum_{i=1}^{r}\alpha_i T_i$ with $T_i = (1-\lambda_i)I + \lambda_i P_{C_i}$ for all i, $0 < \lambda_i < 2$, $\alpha_i \ge 0$ for $i = 0, 1, 2, \ldots, r$, $\sum_{i=0}^{r}\alpha_i = 1$ where $C_0 = \bigcap_{i=1}^{r}C_i$ is nonempty. Then starting from an arbitrary element x of H, the sequence $\{T^nx\}$ converges weakly to an element of C_0. Later, Kitahara and Takahashi [4] and Takahashi and Tamura [10] dealt with the feasibility problem by convex combinations of sunny nonexpansive retractions in uniformly convex Banach spaces.

Using Theorem and Proposition in Section 3, we obtain two results.

Corollary 4.1. *Let E be a uniformly convex Banach space whose norm is uniformly Gâteaux differentiable and let C be a closed convex subset of E. Let $S_1, S_2, \dots, S_r$ be nonexpansive mappings of C into itself such that $\bigcap_{i=1}^r F(S_i) \neq \emptyset$. Define a family of finite nonexpansive mappings $\{T_1, T_2, \dots, T_r\}$ by $T_i = (1-\lambda_i)I + \lambda_i S_i$ for $i = 1, 2, \dots, r$, $0 < \lambda_i < 1$. Let $\{\alpha_n\} \subset (0,1)$ be a sequence which satisfies $\lim_{n\to\infty} \alpha_n = 0$, $\sum_{n=1}^{\infty} \alpha_n = \infty$ and $\sum_{n=1}^{\infty} |\alpha_{n+r} - \alpha_n| < \infty$. Define a sequence $\{x_n\}$ by $x_0 \in C$ and*

$$x_{n+1} = \alpha_{n+1} x_0 + (1-\alpha_{n+1}) T_{n+1} x_n \quad \text{for all } n = 0, 1, 2, \dots,$$

where $T_{n+r} = T_n$. Then $\{x_n\}$ converges strongly to a common fixed point z of $S_1, S_2, \dots, S_r$. Further, if $Px_0 = \lim_{n\to\infty} x_n$ for each $x_0 \in C$, then P is a sunny nonexpansive retraction of C onto $\bigcap_{i=1}^r F(S_i)$.

Proof. By Proposition 3.1 and Theorem 3.2, we have $\{x_n\}$ converges to a common fixed point of $S_1, \dots, S_r$. □

Corollary 4.2. *Let E be a uniformly convex Banach space whose norm is uniformly Gâteaux differentiable and let C be a closed convex subset of E. Let $C_1, C_2, \dots, C_r$ be nonexpansive retracts of C such that $\bigcap_{i=1}^r C_i \neq \emptyset$. Define a family of finite nonexpansive mappings $\{T_1, T_2, \dots, T_r\}$ by $T_i = (1-\lambda_i)I + \lambda_i P_{C_i}$, where $0 < \lambda_i < 1$ and P_{C_i} is a nonexpansive retraction of C onto C_i, for $i = 1, 2, \dots, r$. Let $\{\alpha_n\} \subset (0,1)$ be a sequence which satisfies $\lim_{n\to\infty} \alpha_n = 0$, $\sum_{n=1}^{\infty} \alpha_n = \infty$ and $\sum_{n=1}^{\infty} |\alpha_{n+r} - \alpha_n| < \infty$. Define a sequence $\{x_n\}$ by $x_0 \in C$ and*

$$x_{n+1} = \alpha_{n+1} x_0 + (1-\alpha_{n+1}) T_{n+1} x_n, \ n = 0, 1, 2, \dots,$$

where $T_{n+r} = T_n$. Then $\{x_n\}$ converges strongly to a point z of $\bigcap_{i=1}^r C_i$. Further, if $Px_0 = \lim_{n\to\infty} x_n$ for each $x_0 \in C$, then P is a sunny nonexpansive retraction of C onto $\bigcap_{i=1}^r C_i$.

Proof. By Corollary 4.1 and $\bigcap_{i=1}^r F(P_{C_i}) = \bigcap_{i=1}^r C_i$, we obtain the conclusion. □

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