

## EQUILIBRIUM IN NO-INFORMATION BEST-CHOICE GAMES PLAYED BY EQUALLY WEIGHTED PLAYERS

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**ABSTRACT.** Two players observe a stochastic stream of offers. They arrive via continuous-time simple Markov process in the time interval  $[0, 1]$  and their arrival becomes less probable as time passes. At each arrival, players must decide either accept (A) or reject (R), immediately and independently. Players have equal weights, so if both players want to accept a same offer, a lottery is used to the effect that each player can get it with equal probability  $1/2$ . If one player accepts an offer and the other doesn't the game goes on as one-person game for the latter. A player wins if he accepts the latest offer arriving before time 1. Each player aims to find his strategy that maximizes his probability of winning. The normal form of the game is formulated and the structural form of the solution to this game is found to calculate the Nash equilibrium. Another best-choice games where each player aims to accept an offer later than the opponent is also discussed.

### 1. Introduction and Formulation of the Problem.

This paper deals with some problems of optimal stopping games over continuous-time simple Markov process. The problem considered is related to the No-Information best-choice problem (or, so-called, secretary problem). First we state the problem.

(1°) Two players I and II observe a continuous-time 0-1 valued stochastic process  $x(t)$ ,  $0 \leq t \leq 1$ , which is simple Markov with transition density

$$\Pr.\{x(\tau) = 0, \forall \tau \in (t, s) \text{ and } x(s + \Delta s) - x(s) = 1 \mid x(t) = 1\} = ts^{-2} \Delta s$$

for  $0 < t < s < 1$ , and hence

$$\Pr.\{x(\tau) = 0, \forall \tau \in (t, 1] \mid x(t) = 1\} = 1 - \int_t^1 ts^{-2} ds = t.$$

(2°) We call the event  $x(t) = 1$ , as “an offer arrives at time  $t$ .” At each arrival of an offer, both players choose, simultaneously and independently, either to accept (A) or to reject (R) the offer. If I-II choice-pair is A-R(R-A) then I(II) accepts the offer and drops out hereafter from the game, and his opponent continues his one-person game. If I-II choice is A-A then a lottery (A-R, R-A;  $1/2, 1/2$ ) is used to the effect that A-R or R-A is enforced to the players with probability  $1/2$  each. If I-II choice is R-R, then the offer is rejected and the players face the next arrival of an offer.

(3°) A player wins if he accepts the latest offer arriving in the period  $0 \leq t \leq 1$ . Each player aims to find his strategy by following which he maximizes his probability of winning.

Suppose two job-searchers seek for a job opportunity. Employer offers jobs sequentially one-by-one. The arriving offers are competed to accept by them, with the aim of obtaining the absolutely best offer. Suppose the total number  $N$  of the offers is fixed and known,

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and the  $n$ -th offer arrived is relatively best (*i.e.*, best among those seen so far). Then the probabilistic manner of the ratio  $t = \frac{n}{N} \in [0, 1]$  is described asymptotically (*i.e.*,  $n, N \rightarrow \infty$ ) by the equations in (1°).

Define state  $((t))$  to mean that both players remain in the game and an offer has just arrived at time  $t$ . Also define state  $(t)$  to mean that one player has just dropped out from the game at time  $t$ .

Let  $U(t)$  be the optimal probability of winning for the remaining player in state  $(t)$ . Then it satisfies the integral equation

$$(1.1) \quad U(t) = \int_t^1 (t/s^2)(s \vee U(s))ds \quad (0 < t \leq 1; \quad U(1) = 0)$$

Considering symmetry in the players role in the game, let  $V(t)$  be the equilibrium probability of winning for each player in state  $((t))$ .

Then we have the Optimality Equation

$$(1.2) \quad (V(t), V(t)) = \text{Eq.Val. } M(t, V(t), V(t)) \quad (0 < t \leq 1; \quad V(1) = 1/2)$$

where

$$(1.3) \quad M(t, V(t), V(t)) = \begin{array}{cc} & \begin{array}{cc} \text{R} & \text{A} \end{array} \\ \begin{array}{c} \text{R} \\ \text{A} \end{array} & \begin{array}{|c|c|} \hline TV(t), TV(t) & U(t), t \\ \hline t, U(t) & \frac{1}{2}(t + U(t)), \frac{1}{2}(t + U(t)) \\ \hline \end{array} \end{array}$$

and

$$TV(t) = t \int_t^1 s^{-2}V(s)ds,$$

*i.e.*,  $T$  is a transition operator. As to the interpretation of Eq.Val. in (1.2), see Remark 4 in Section 4.

Optimal stopping games have various phases according to the various essentials of (1°) zero-sum or nonzero-sum, (2°) information pattern under which players choose their decision, and (3°) the objectives that players attempt. The problem we consider in this paper belongs to a class of no-information best-choice problem combined with continuous-time sequential games. Recent works related to this aeria of problems are [2, 3, 5, 6, 7]. Ref.[2, 3, 7] discusse discrete-time sequential games. Ref.[5, 6] analyse continuous-time sequential games where offers arrive in a Poison manner. Also a recent look for the optimal stopping games in various phases can be found in Ref.[4].

**2. Selecting Latest Offer.**

**Lemma 1.1** *The solution to (1.1) is*

$$(2.1) \quad U(t) = \begin{cases} e^{-1}, & \text{if } 0 < t < e^{-1} \\ -t \log t, & \text{if } e^{-1} < t \leq 1. \end{cases}$$

*The optimal strategy in state  $(t)$  is : Accept the earliest offer (if any) arriving after time  $e^{-1}$ .*

*Proof.* By letting  $u(t) = t^{-1}U(t)$ , (1.1) becomes

$$u(t) = \int_t^1 s^{-1}(1 \vee u(s))ds$$

and hence

$$(2.2) \quad u'(t) = -t^{-1}(1 \vee u(t)), \quad \text{with } u(1) = U(1) = 0.$$

Integration of  $u'(t) = -t^{-1}$ , for  $t_1 < t \leq 1$ , with  $u(1) = 0$ , gives  $t_1 = e^{-1}$  and  $u(t) = -\log t$ ,  $t_1 < t \leq 1$ . Since  $u(t)$  is decreasing in  $t$  from (2.2),

$$u'(t) = -t^{-1}u(t), \quad 0 < t < t_1, \quad \text{with } u(t_1) = 1,$$

and we get  $u(t) = e^{-1}t^{-1}$ . Thus

$$u(t) = \begin{cases} e^{-1}t^{-1}, & \text{if } 0 < t < e^{-1} \\ -\log t, & \text{if } e^{-1} < t \leq 1. \end{cases}$$

which gives (2.1).  $\square$

Let  $U_0(t)$  be the optimal probability (in the one-person game) of selecting the latest offer, when an offer has just arrived at time  $t$ . Then we have the Optimality Equation

$$U_0(t) = t \vee TU_0(t), \quad 0 < t \leq 1, \quad U_0(1) = 1,$$

which gives

$$(2.3) \quad U_0(t) = t \vee e^{-1},$$

a well-known result in the secretary problem. (Ref.[3]). Note that  $U_0(t)$  is related to  $U(t)$ , by the equality  $TU_0(t) = U(t)$ ,  $0 < t \leq 1$ .

Now the relations  $V(t) \leq 1/2$  and  $TV(t) \leq U(t)$  are evident. The second inequality is based on the definitions of  $V(t)$  and  $U(t)$  and our common sense. Consider the following three cases.

Case 1.  $U(t) < t$ , i.e.,  $e^{-1} < t < 1$ ,

Case 2.  $TV(t) < t < e^{-1} (\equiv t_1)$ ,

Case 3.  $0 < t < TV(t)$ .

**Lemma 1.2** *The common equilibrium value of the bimatrix game  $M(t, V(t), V(t))$  is equal to ;*

$$\begin{aligned} & \frac{1}{2}(t - t \log t), \quad \text{in Case 1;} \\ & \frac{1}{2}(t + t_1) - \frac{1}{4}(t - t_1)^2 \left\{ \frac{1}{2}(t + t_1) - TV(t) \right\}^{-1}, \quad \text{in Case 2;} \\ & TV(t), \quad \text{in Case 3;} \end{aligned}$$

*Equilibrium in the state  $((t))$  is in A-A, Mix-Mix, and R-R in Cases 1,2 and 3, respectively. Here Mix means the mixed-strategy that randomize R and A with probability  $(t - TV(t)) / \{ \frac{1}{2}(t + t_1) - TV(t) \}$ , for A.*

*Proof.* Cases 1 and 3 are evident. So we prove Case 2 only. In Case 2, the two pure-strategy pairs A-R and R-A and one mixed-strategy pair are in equilibrium. ( See Remark 4 in Section 4.) The latter is derived from

$$\begin{aligned} \bar{\varphi}TV(t) + \varphi t_1 &= \bar{\varphi}t + \varphi \frac{1}{2}(t + t_1) = \text{Eq. Payoff to II,} \\ \bar{\psi}TV(t) + \psi t_1 &= \bar{\psi}t + \psi \frac{1}{2}(t + t_1) = \text{Eq. Payoff to I,} \end{aligned}$$

where  $\langle \bar{\varphi}, \varphi \rangle$  and  $\langle \bar{\psi}, \psi \rangle$  represent the mixed-strategy for I and II, respectively. Thus we obtain

$$(2.4) \quad \varphi^* = \psi^* = \frac{t - TV(t)}{\frac{1}{2}(t + t_1) - TV(t)},$$

and the common eq. payoff

$$\begin{aligned} \bar{\varphi}^*t + \varphi^* \frac{1}{2}(t + t_1) &= \left\{ tt_1 - \frac{1}{2}(t + t_1)TV(t) \right\} / \left\{ \frac{1}{2}(t + t_1) - TV(t) \right\} \\ &= \frac{1}{2}(t + t_1) - \frac{1}{4}(t - t_1)^2 \left\{ \frac{1}{2}(t + t_1) - TV(t) \right\}^{-1}. \quad \square \end{aligned}$$

Applying Lemma 1.1 and 1.2 to (1.2), we get

**Theorem 1.** *The common eq. value  $V(t)$ , for our problem (1.1) ~ (1.3) is as follows : Determine  $t_2 \in (0, t_1)$  by the equation*

$$(2.5) \quad V(t_2) = t_2,$$

where  $V(t)$ ,  $t_2 < t < t_1$  satisfies the integral equation

$$(2.6) \quad TV(t) = \frac{1}{2}(t + t_1) - \frac{1}{4}(t - t_1)^2 \left\{ \frac{1}{2}(t + t_1) - V(t) \right\}^{-1},$$

with  $V(t_1) = t_1$  and  $TV(t_1) = (3/4)t_1$ .

Then

$$(2.7) \quad V(t) = \begin{cases} \frac{1}{2}(t - t \log t), & \text{for } t_1 < t \leq 1, \\ t_2, & \text{for } 0 < t < t_2. \end{cases}$$

Common eq. strategy is:

Condition	Common eq. strategy
$0 < t < t_2$	Choose R
$t_2 < t < t_1$	Randomize R and A with prob. $\varphi(t) = 2(V(t) - t)/(t_1 - t)$ , for A.
$t_1 < t < 1$	Choose A

in state  $((t))$ , and the optimal strategy in state  $(t)$  is : Accept the earliest offer, if any, that arrives after time  $t \vee e^{-1}$ ,

*Proof.* In Case 1,  $V(t) = \frac{1}{2}(t - t \log t)$ , which is concavely increasing in  $t_1 < t \leq 1$ , with values  $t_1$  at  $t = t_1$  and  $1/2$  at  $t = 1$ . Also

$$(2.8) \quad TV(t) = t \int_t^1 s^{-2}V(s)ds = \frac{1}{2}(-t \log t) + \frac{1}{4}t(\log t)^2$$

is concavely decreasing with values  $(3/4)t_1$  at  $t = t_1$  and  $0$  at  $t = 1$ .

In Case 2, we have

$$V(t) = \frac{1}{2}(t + t_1) - \frac{1}{4}(t - t_1)^2 \left\{ \frac{1}{2}(t + t_1) - TV(t) \right\}^{-1}$$

which, after a rearrangement, becomes the integral equation (2.6). This shows that  $V(t)$  and  $TV(t)$  are interchangeable. Substituting (2.6) into (2.4) we easily find that

$$(2.4') \quad \varphi^* = \psi^* = \frac{t - TV(t)}{\frac{1}{2}(t + t_1) - TV(t)} = \frac{2(V(t) - t)}{t_1 - t}.$$

This function of  $t$ ,  $t_2 < t < t_1$ , satisfies  $\varphi^*(t_2) = 0$  and  $\varphi^*(t_1) = \lim_{t \rightarrow t_1} 2(1 - V'(t)) = 2(1 + \frac{1}{2} \log t_1) = 1$ .

The condition required by  $t_2$  is  $TV(t_2) = t_2$ , which by using (2.6) gives  $V(t_2) = t_2$ , *i.e.*, (2.5). Finally

$$V(t) = TV(t), \quad \text{for } 0 < t < t_2$$

becomes, by taking  $v(t) = t^{-1}V(t)$ ,  $v(t) = \int_t^1 s^{-1}v(s)ds$ . Hence  $v'(t)/v(t) = -t^{-1}$ , and, by integration over  $(t, t_2)$ ,  $v(t) = t_2v(t_2)t^{-1}$ . Therefore  $V(t) = \text{const.}$ , that is  $V(t) = V(t_2) = t_2$  by (2.5).  $\square$

**Remark 1.** Comparing the two-person game with the corresponding one-person game we note that  $U_0(0+) = e^{-1} > t_2 = V(0+)$ , from (2.3) and (2.7).

**Remark 2.** In order to find the value of  $t_2$ . We must derive the solution of (2.6) explicitly. See Remark 3 in Section 3 and Figure 1(a) in Section 4.

**3. Selecting the Latest Offer or an Offer Later Than the Opponent.**

We consider in this section the game where the rule is : (1°) and (2°) are the same as in Section 1, but (3°) is replaced by

(3<sup>+</sup>) A player wins if he gets the latest offer or if he gets an offer later than his opponent. Each player aims to maximize his probability of winning.

The analysis of the game is mostly the same as in Section 2. The Optimality Equation is

$$(3.1) \quad (V(t), V(t)) = \text{Eq.Val. } (K(t, V(t), V(t)), \quad (0 < t \leq 1, V(1) = 1/2)$$

where

$$(1.3) \quad K(t, V(t), V(t)) = \begin{array}{c} \text{R} \\ \text{A} \end{array} \begin{array}{|c|c|} \hline \text{R} & \begin{array}{c} \text{R} \\ \text{A} \end{array} \\ \hline \text{TV}(t), \text{TV}(t) & \bar{t}, t \\ \hline t, \bar{t} & 1/2, 1/2 \\ \hline \end{array}$$

since  $\text{Pr.}\{\text{an offer arrives after time } t \mid \text{state } (t)\} = t \int_t^1 s^{-2} ds = 1 - t$ .

The relations  $V(\bar{t}) \leq 1/2$  and  $TV(t) \leq \bar{t}$  are evident. The latter inequality is based on our common sense.

**Lemma 2.1** *The common eq. value of  $K(t, V(t), V(t))$  is equal to :*

$$\begin{cases} 1/2, & \text{if } \frac{1}{2} < t < 1, \text{ (A-A is in eq.)}; \\ \frac{1}{2} - (\frac{1}{4} - t\bar{t}) (\frac{1}{2} - TV(t))^{-1}, & \text{if } TV(t) < t < \frac{1}{2}, \text{ (Mix-Mix is in eq.)}; \\ TV(t), & \text{if } 0 < t < TV(t), \text{ (R-R is in eq.)}. \end{cases}$$

Here Mix for each player means randomization between R and A with probability

$$(3.3) \quad \bar{\varphi}(t) = \left(\frac{1}{2} - t\right) \left(\frac{1}{2} - TV(t)\right)^{-1}, \text{ for } R.$$

*Proof.* Consider the three cases

Case 1.  $TV(t) \leq \bar{t} < \frac{1}{2} < t$ .

Case 2.  $TV(t) \leq t < \frac{1}{2} < \bar{t}$ .

Case 3.  $t < \frac{1}{2} \wedge TV(t) = TV(t)$ . ( $\because V(t) \leq \frac{1}{2}$  gives  $TV(t) \leq \frac{1}{2}\bar{t}$ ).

The proof is made similarly as in Lemma 1.2. □

**Theorem 2.** *The common eq. value  $V(t)$ , for our problem (3.1)-(3.2) is as follows: Determine  $t_2 \in (0, \frac{1}{2})$  by the equation*

$$(3.4) \quad V(t_2) = t_2,$$

where  $V(t)$ ,  $t_2 < t < 1/2$  satisfies the integral equation

$$(3.5) \quad TV(t) = \frac{1}{2} - \left(\frac{1}{4} - t\bar{t}\right) \left(\frac{1}{2} - V(t)\right)^{-1}, \text{ with } V(1/2) = 1/2.$$

Then for  $t \in (0, t_2) \cup (1/2, 1]$

$$(3.6) \quad V(t) = \begin{cases} 1/2, & \text{for } 1/2 < t \leq 1 \\ t_2, & \text{for } 0 < t < t_2. \end{cases}$$

*Common eq. strategy is:*

Condition $0 < t < t_2$	Choose R
$t_2 < t < 1/2$	Randomize R and A with prob. $\bar{\varphi}(t) = \frac{1/2 - V(t)}{1/2 - t}$ , for R.
$1/2 < t < 1$	Choose A

in state  $(t)$ , and the optimal strategy in state  $(t)$  is: Accept the earliest offer which arrives after time  $t$ .

*Proof.* For  $\frac{1}{2} < t \leq 1$ , we have  $V(t) = \frac{1}{2}$ , and so  $TV = \bar{t}/2$ . For  $t$  such that  $TV < t < \frac{1}{2}$ , we have

$$(3.7) \quad V(t) = \frac{1}{2} - \left(\frac{1}{4} - t\bar{t}\right) \left(\frac{1}{2} - TV(t)\right)^{-1}$$

which, after a rearrangement, becomes the differential equation (3.5). So,  $V(t)$  and  $TV(t)$  are again interchangeable.

Substituting (3.5) into (3.3) we easily find that

$$(3.3') \quad \bar{\varphi} = \frac{1/2 - t}{1/2 - TV(t)} = \frac{1/2 - V(t)}{1/2 - t}.$$

This function of  $t$ , for  $t_2 < t < 1/2$ , satisfies

$$\bar{\varphi}(t_2) = 1 \quad \text{and} \quad \bar{\varphi}\left(\frac{1}{2} + 0\right) = V'\left(\frac{1}{2} + 0\right) = 0.$$

The condition required by  $t_2$  is  $TV(t_2) = t_2$ , which, by using (3.5), gives  $V(t_2) = t_2$ , i.e., (3.4). The rest of the proof is the same as in Theorem 1.  $\square$

**Corollary 2.1.** *The solution to the integral equation (3.5) is*

$$(3.8) \quad V(t) = \frac{1}{2} - \left(\frac{1}{2} - t\right)^2 \left[ \left\{ \frac{13}{4} - 2\log(2t) \right\} t^2 - 2t + \frac{1}{4} \right]^{-1/2}$$

or equivalently

$$TV(t) = \frac{1}{2} - \left[ \left\{ \frac{13}{4} - 2\log(2t) \right\} t^2 - 2t + \frac{1}{4} \right]^{1/2}. \quad (t_2 \leq t \leq 1/2; \quad V(1/2) = 1/2)$$

and  $t_2 \cong 0.3151$  is a unique root in  $(0, 1/2)$  of the equation

$$(3.9) \quad -t \log t = \frac{1}{2} - \left(\frac{9}{8} - \log 2\right) t,$$

*Proof.* Let, for  $t_2 < t < 1/2$ ,

$$w(t) \equiv \int_t^{1/2} (s^{-2}V(s) - s^{-1})ds = \int_t^{1/2} s^{-2}V(s)ds + \log(2t).$$

Then, since

$$V(t) = t - t^2w'(t) \quad \text{and} \quad TV(t) = t \{w(t) - \log(2t) + 1/2\},$$

Eq.(3.5) becomes the differential equation

$$(3.10) \quad \log(2e^{-1/2}) + \frac{1}{2t} + \log t - w(t) = \frac{t^{-3}(1/2 - t)^2}{(2t^2)^{-1} - t^{-1} + w'(t)}$$

i.e.,

$$-\frac{1}{2} \frac{d}{dt} \left[ \left\{ \log(2e^{-1/2}) + \frac{1}{2t} + \log t - w(t) \right\}^2 \right] = t^{-3}(1/2 - t)^2$$

Integrating both sides with respect to  $t \in (t_2, 1/2)$ , we obtain

$$\log(2e^{-1/2}) + \frac{1}{2t} + \log t - w(t) = \left[ \frac{13}{4} - 2\log(2t) - 2t^{-1} + \frac{1}{4}t^{-2} \right]^{1/2}.$$

Differentiation and using  $V(t) = t - t^2w'(t)$ , finally give (3.8).

Substitute (3.4) into (3.8), then it follows that

$$\left( \frac{1}{2} - t \right)^2 = \left( \frac{13}{4} - 2\log(2t) \right) t^2 - 2t + \frac{1}{4},$$

which becomes (3.9). It is easy to ascertain that the inside of [ ... ] in (3.8) is positive for  $t_2 < t < 1/2$ . This completes the proof.  $\square$

The solution is numerically derived for some values of  $t \in (t_1, t_2)$  from (3.8) and (3.3').

	$V(t)$	$TV(t)$	$\overline{\varphi^*}(y)$ , i.e., prob.for $R$
$t = 0.3151$	0.3151	0.3151	1
0.35	0.3806	0.3116	0.796
0.40	0.4509	0.2965	0.491
0.45	0.4889	0.2746	0.222
0.5	0.5	0.25	0

**Remark 3.** Unfortunately we could not find an explicit solution to the integral equation (2.6) by taking along the same way as in Corollary 2.1. The differential equation corresponding to (3.10), in this case, is

$$\frac{(4t)^{-1}(t_1/t - 1)^2}{(2t)^{-1}(t_1/t - 1) + w'(t)} = 3/4 + t_1/(2t) + \log t - w(t) \quad (t_2 < t < t_1; w(t_1) = 0)$$

where  $w(t) \equiv \int_t^{t_1} (s^{-2}V(s) - s^{-1})ds$ . This equation becomes another simpler one

$$tz' = \sqrt{z} - \frac{1}{2}(t_1/t - 1)^2 \quad (t_2 < t < t_1; z(t_1) = 1/16),$$

if we consider

$$z(t) \equiv \left[ \frac{3}{4} + \frac{t_1}{2t} + \log t - w(t) \right]^2 = \left[ -\frac{1}{4} + \frac{t_1}{2t} - \int_t^{t_1} s^{-2}V(s)ds \right]^2.$$

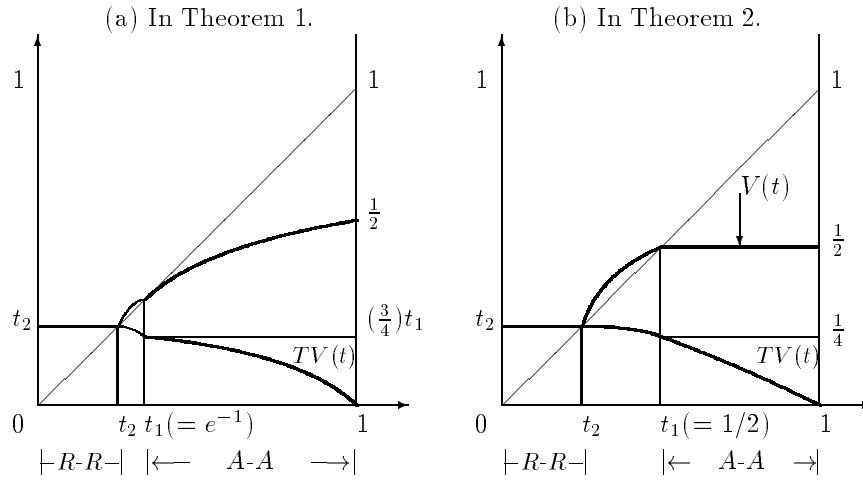
**4. Final Remarks.**

**Remark 4.** For the bimatrix game (1.3) in Lemma 1.2. and (3.2) in Lemma 2.1. (i) two pure-strategy pairs and one mixed-strategy pair are in eq. when  $TV(t) < t < t_1$ . We assume that we take (ii) in order to make eq. strategies and eq. values continuous in  $t \in (0, 1]$ . Thus Eq. Val. in Optimality Equations (1.2) and (3.1) is well-defined in this sense.

**Remark 5.** The condition that  $\varphi^*$ , given by (2.4)-(2.4') in Theorem 1 (given by (3.3)-(3.3') in Theorem 2), satisfies  $0 \leq \varphi^* \leq 1$ , is equivalent to the relation  $TV(t) < t < V(t)$ . Proof is easy and omitted. Common eq. values  $V(t)$  in the two games in Section 2 and 3 are shown by Figure 1(a)-1(b). Whether  $t_2 < \frac{3}{4}e^{-1}$  in (a) is true or not is unknown, although the fact that  $t_2 \doteq 0.3151$  in (b) is greater than  $t_2$  in (a), is evident.



Figure 1. Common eq. values  $V(t)$  in the two games.



The eq. strategy pairs in state  $((t))$  are also mentioned. In the interval  $(t_2, t_1)$ , they are Mix-Mix, given in Theorems 1 and 2. The broken curve in (a) is one which is supposed to be.

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**Addendum.** After this article was put into publication, an article:

P. Neumann, D. Ramsey, and K. Szajowski, Randomized stopping times in Dynkin games, *to appear in Z. Angew. Math. Mech.*

came to the author's attention. The authors of this paper investigate deeply on the problem in Section 2 of the present article and suggest that  $t_2$ , in Theorem 1, is approximately 0.2885 by using a high-speed computer.

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