CORRECTION TO THE PAPER "THERE ARE NO CODIMENSION 1 LINEAR ISOMETRIES ON THE BALL AND POLYDISK ALGEBRAS"

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Abstract. The following is Theorem in [1]. Let A be the ball algebra or the polydisk algebra in $\mathbb{C}^n$. When $n > 1$, there are no codimension 1 linear isometries on A.

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Professor Junzo Wada kindly indicated the author an error in the proof of Theorem for the case of the polydisk algebra in [1]. The purpose of this note is to supply a correct proof of the result. Let D^n be the unit polydisk, $\bar{D}^n$ the closure, ∂D^n the topological boundary and T^n the torus. Let $A(T^n)$ be the polydisk algebra.

Theorem When $n > 1$, there are no codimension 1 linear isometries on $A(T^n)$.

The error in the proof in [1] is the last part (from the line 5 from the bottom on page 389 to the line 10 from the top on page 390).

We will simply write $A = A(T^n)$. Suppose that there exists a codimension 1 linear isometry, say $T : A \to A$. We will show a contradiction. By [1], there exists a homeomorphism φ of T^n onto T^n and a function $\psi \in A$ such that $|\psi(x)| = 1$ for all $x \in T^n$, and

$$(Tf)(x) = \psi(x)f(\varphi(x)) \text{ for all } x \in T^n \text{ and all } f \in A.$$

We denote by $\hat{f}$ the Gelfand transform of $f \in A$ and $\hat{A}$ the Gelfand transform of A. We identify $\hat{A}$ with the polydisk algebra on $\bar{D}^n$. We define an operator $\hat{T}$ on $\hat{A}$ by

$$\hat{T}\hat{f} = \widehat{Tf},\ \hat{f} \in \hat{A}.$$

Since $A \to \hat{A}$ is an isometry, $\hat{T}$ is a codimension 1 linear isometry on $\hat{A}$.

So, by a simple caluculation, we see that $\hat{\psi}$ has a unique zero $p = (p_1, p_2, \dots, p_n) \in \bar{D}^n \backslash T^n$. Since $\hat{\psi}$ is holomorphic on D^n, we have $p \in \partial D^n \backslash T^n$. Thus there exists $1 \le j \le n$ such that $|p_j| < 1$. Put

$$U = \{(z_1, \dots, z_n) \in \bar{D}^n : |z_j - p_j| < \frac{1}{2}(1 - |p_j|) \text{ and } z_l = p_l \text{ for } 1 \le l \le n,\ l \ne j\}$$

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and

$$U_k = \{(z_1, \dots, z_n) \in \bar{D}^n : |z_j - p_j| < \frac{1}{2}(1 - |p_j|)$$
$$\text{and } z_l = \Big(1 - \frac{1}{k}\Big)p_l \text{ for } 1 \le l \le n,\ l \ne j\}$$

Then $\hat{\psi}|_{U_k}$ is holomorphic on U_k and has no zero. On the other hand, $\hat{\psi}|_U$ is holomorphic on U and has a unique zero. By Rouché's theorem, this is a contradiction.

References

[1] K. Kasuga, *There are no codimension 1 linear isometries on the ball and polydisk algebras,* Sci. Math. Japonicae **54** No.2 (2001), 387-390.

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