Furuta inequality and Gan-Liu-Tam inequality: Simple proofs

Dedicated to the 100-th birthday of Professor Zirô Takeda

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ABSTRACT. From the viewpoint of sequential product, we present a simple proof to a satellite of Furuta inequality, i.e., if $A \geq B \geq 0$, then for each $r \geq 0$,

$$A^{-r} \#_{\frac{1+r}{p+r}} B^p \leq B \ (\leq A)$$

holds for $p \geq 1$ and $r \geq 0$, where $\#_t$ for $t \in [0,1]$ is the operator geometric mean with weight t in the sense of Kubo-Ando. We also do it to Gan-Liu-Tam inequality on log-majorization.