Satisfiability degrees for BCK-algebras

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Abstract. We investigate the satisfiability degree of some equations in finite BCK-algebras; that is, given a finite BCK-algebra and an equation in the language of BCK-algebras, what is the probability that elements chosen uniformly randomly with replacement satisfy that equation?

Specifically we consider the equations for the excluded middle, double negation, commutativity, positive implicativity, and implicativity. We give a sufficient condition for an equation to have a finite satisfiability gap among commutative BCK-algebras, and prove that the law of the excluded middle has a gap of $\frac{1}{3}$, while the positive implicative and implicative equations have gap $\frac{1}{9}$. More generally, though, in the language of BCK-algebras, we show that double negation, commutativity, positive implicativity, and implicativity all fail to have finite satisfiability gap. We provide bounds for the probabilities in these cases.